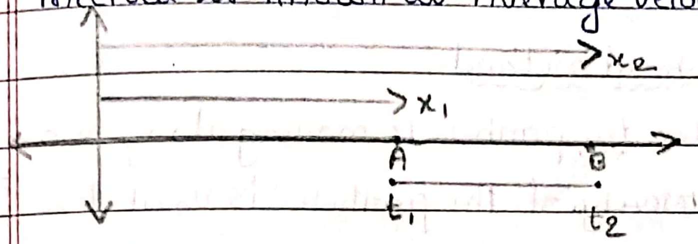
3. motion in a [redacted] Straight line revesion:

1. SAQ's:

★★★ 1. Explain the term's averagevelocity and instantaneous velocity and when the are equal?

Sol: **Averagevelocity**:- The ratio of displacement to the time interval is Known as Average velocity.

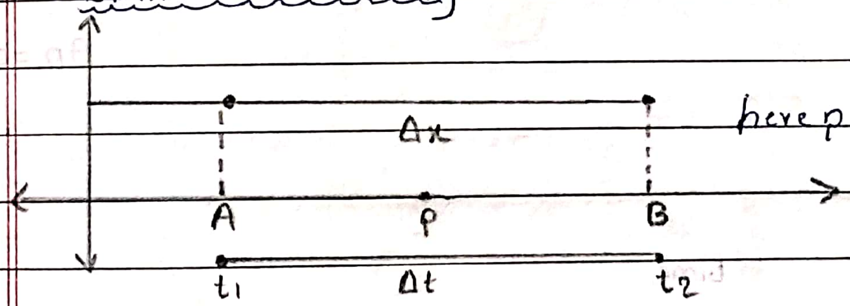


here $\Delta t = t_2 - t_1$

$$\text{averagevelocity} = \frac{\text{displacement}}{\text{time interval}}$$

$$V_x = \frac{\Delta x}{\Delta t} \Rightarrow \boxed{V_x = \frac{x_2 - x_1}{t_2 - t_1}}$$

Instantaneousvelocity:



here p = particular instant

The velocity of a particle at a particular instant of time is Known as instantaneous velocity. Instantaneous velocity is the limiting value of averagevelocity.

$$V_x = \frac{\Delta x}{\Delta t} \rightarrow \text{limiting value of averagevelocity}$$

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$\boxed{V = \frac{dx}{dt}}$$

for uniform motion average velocity and instantaneous velocity are equal

★★★ 2. Derive the equation $s = ut + \frac{1}{2}at^2$ using graphical method?

Sol: ⇒ using graphical method :-

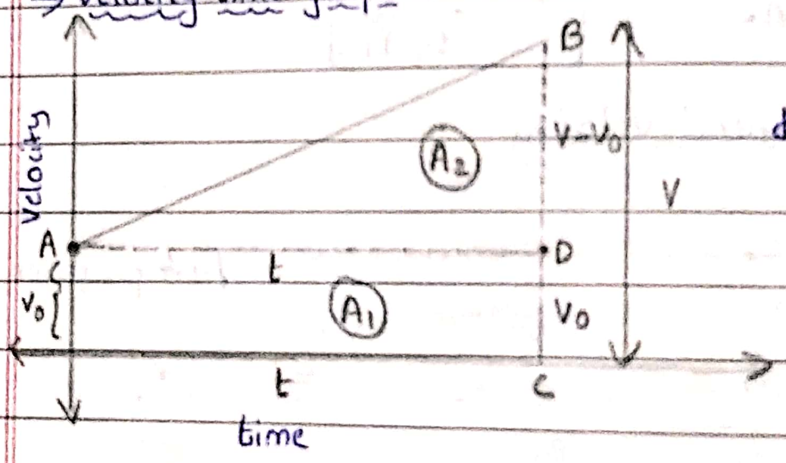
> let us consider the particle is moving along x axis

> The initial velocity of the particle is v_0 at $t = 0$ seconds

> The final velocity of the particle is v at t seconds

> The particle is moving with constant acceleration

⇒ velocity time graph :



thus $OA = DC = v_0$

$AD = OC = t$

> OA represents the initial velocity v_0

> OC represents the final velocity v

⇒ displacement = area under the curve

∴ displacement = area of rectangle + area of traingle

$\boxed{x = A_1 + A_2}$ —(1)

∴ $A_1 = OC \times OA$ {length x breadth}

$\boxed{A_1 = v_0 t}$

$A_2 = \frac{1}{2} \times AB \times BD$ here from equation of motion

$A_2 = \frac{1}{2} t \times [v - v_0]$ $V = u + at$

$\therefore A_2 = \frac{1}{2} t [at]$ $V = V_0 + at$

$v - v_0 = at$

$$\boxed{A_2 = \frac{1}{2} at^2}$$

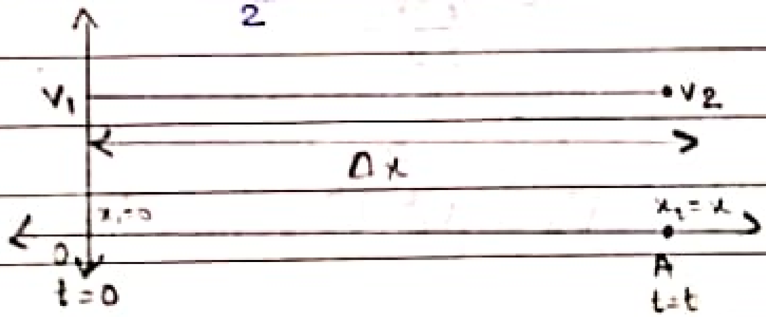
from equation —(1)

$x = A_1 + A_2$

$$\boxed{x = v_0 t + \frac{1}{2} at^2}$$ hence proved

★★ 3. A particle moves in straight line with uniform acceleration. It's velocity at $t=0$ is v_1 and at $t=t$ is v_2 then show that the average velocity of that particle is $\frac{v_1+v_2}{2}$

Sol:



Yes, it is correct.

average velocity = $\frac{\text{displacement}}{\text{time interval}}$

$$V_x = \frac{\Delta x}{\Delta t}$$

$$V_x = \frac{x_2 - x_1}{t_2 - t_1}$$

$$V_x = \frac{x - 0}{t - 0}$$

$$\boxed{V_x = \frac{x}{t}} \quad \text{—(1)}$$

from the equation of motion

$$v^2-u^2=2as$$

$$v_2^2-v_1^2=2ax$$

$$x=\frac{v_2^2-v_1^2}{2a}$$

from the equation of motion

$$v=u+at$$

$$v_2=v_1+at$$

$$t=\frac{v_2-v_1}{a}$$

from the equation - ①

$$v_x=\frac{x}{t}$$

$$v_x=\frac{\frac{v_2^2-v_1^2}{2\cancel{a}}}{\frac{v_2-v_1}{\cancel{a}}}$$

$$v_x=\frac{v_2^2-v_1^2}{(v_2-v_1)2}$$

$$v_x=\frac{\cancel{(v_2-v_1)}(v_2+v_1)}{2\cancel{(v_2-v_1)}}$$

$$\therefore \boxed{v_x=\frac{v_1+v_2}{2}}$$ hence proved.

★★ 4. A vehicle travel's half of the distance L with speed v_1 and another half with speed v_2 and what is average speed

Sol: Given

total distance = L

total time = $t_1 + t_2$

then average speed = $\frac{\text{total distance}}{\text{total time}}$

$$\boxed{V_x = \frac{L}{t_1+t_2}} \quad \text{---(1)}$$

as $V = \frac{x}{t}$

$\boxed{x = vt}$

$\therefore \frac{L}{2} = v_1 t_1$ and $\frac{L}{2} = v_2 t_2$

$t_1 = \frac{L}{2v_1}$ $\qquad$ $t_2 = \frac{L}{2v_2}$

Substituting t_1 and t_2 values in equation - (1)

$$V_x = \frac{L}{\frac{L}{2v_1} + \frac{L}{2v_2}}$$

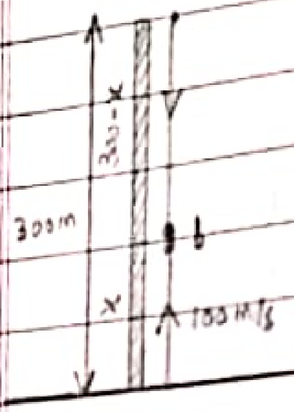
$$V_x = \frac{\cancel{L}}{\cancel{L}\left[\frac{1}{2v_1} + \frac{1}{2v_2}\right]}$$

$$V_x = \frac{1}{\frac{v_1+v_2}{2v_1v_2}}$$

$$\boxed{V_x = \frac{2v_1v_2}{v_1+v_2}}$$ hence proved

~~AIEE~~

★★ 5. A body is dropped from a top of tower of a height 300 m at the same time another ball is thrown vertically upward's with a velocity of 100 m/s. When and where the two bodies meet?



$h = 300m$

$a = 100\ m/s$

$\Rightarrow$ Step (i)

> The vertically projected body

$S = ut + \frac{1}{2}at^2$

here $S = x \Rightarrow 100 = u$

$a = -g$

$\therefore \quad x = 100t + \frac{1}{2}[-g]t^2$

$x = 100t - \frac{1}{2}gt^2 \quad — (1)$

$\Rightarrow$ Step (ii)

for freely falling body

$S = ut + \frac{1}{2}at^2$

$S = 300 - x,\ u = 0$

$a = g$

$300 - x = 0 \cdot t + \frac{1}{2}at^2$

$300 - x = \frac{1}{2}gt^2$

from the equation — (1)

$x = 100t - \frac{1}{2}gt^2$

$x = 100t - [300 - x]$

$\cancel{x} = 100t - 300 + \cancel{x}$

$t = 300/100$

$\boxed{t = 3 \text{ seconds}}$

again from the equation -(1)

$x = 100 - \frac{1}{2}gt^2$

here $t = 3$, $g = 9.8\ m/s^2$

$\therefore x = 100 \cdot 3 - \frac{1}{\cancel{2}} \overset{4.9}{9.8} [3]^2$

$x = 300 - 4.9 \times 9$

$x = 300 - 44.1$

$\boxed{x = 255.9\ m}$

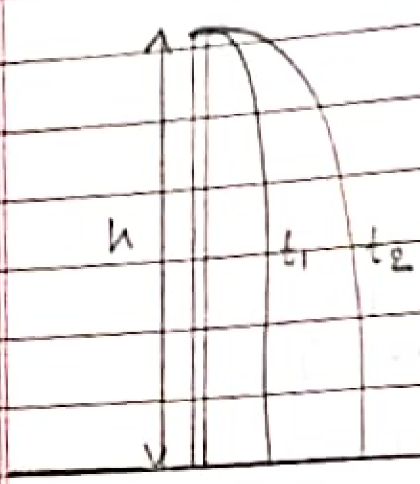
★6. when the velocity of body be in one direction other than the acceleration of body if so give example.

Sol: Yes, it is possible for the body to have velocity in one direction other than acceleration.

example:- For a body projected vertically upwards before reaching the highest point in this case the velocity of body is directed vertically upwards which decreases in magnitude due to the acceleration due to gravity which is always directed downwards Thus the velocity and acceleration are in opposite direction.

★★7. The ball is dropped from the roof of tall buiding Simultaneously other ball is thrown horizontally with Same velocity at from same point which ball lands first. explain your answer.

Sol:



$t_1^2 = \frac{2h}{g}$

$h = \sqrt{\frac{2h}{g}}$

$t_1 = \sqrt{\frac{2h}{g}}$ —①

as $u = u\cos\theta$

$u = 0$

as $s = ut + \frac{1}{2}at^2$

$s = h, u = 0, t = t_2, a = g$

$h = 0.t + \frac{1}{2}gt_2^2$

$\frac{2h}{g} = t_2^2$

$t_2 = \sqrt{\frac{2h}{g}}$ —②

from the equation's ① and ②

$\boxed{t_1 = t_2}$

Therefore both the bodies touch the ground at same time.

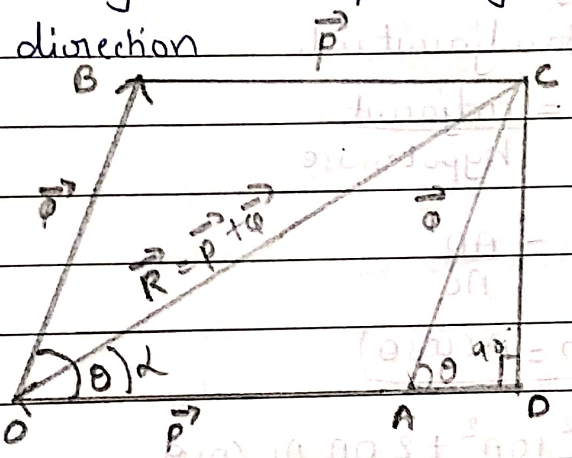
4. Motion in a plane revesion

I SAQ's

★★★ 1. State parellogram law of vector's and expression for magnitude and direction

Sol':

parellogram law of vector's :- If two vector's represent the adjacent sides of a parellogram both in magnitude and direction then their resultant is represented with diagonal of the parellogram both in magnitude and direction



→ expression for magnitude:

from the diagram
$OB = AC = \vec{Q}$
$BC = OA = \vec{P}$

∴ The magnitude of $\vec{OA} = |\vec{OA}|$
$= |\vec{P}|$
$= P$

The magnitude of $\vec{OB} = |\vec{OB}|$
$= |\vec{Q}|$
$= Q$

The magnitude of $OC = R$

from the traingle ΔOCD

$$OC^2 = CD^2 + OD^2$$

here $OD = OA + AD$

$$OC^2 = CD^2 + (OA+AD)^2$$

$$OC^2 = CD^2 + OA^2 + AD^2 + 2OA\,AD \quad -(1)$$

from ΔACD

$$AC^2 = CD^2 + AD^2$$

$$\therefore OC^2 = AC^2 + OA^2 + 2(OA)(AD)$$

from traingle ACD

θ is to adjacent side

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{AD}{AC}$$

$$\boxed{AD = AC\cos\theta}$$

$$\therefore OC^2 = AC^2 + OA^2 + 2OA\,AC\cos\theta$$

here $OC = R$, $OA = p$, $AC = \varphi$

$$\therefore R^2 = p^2 + \varphi^2 + 2p\varphi\cos\theta$$

$$\boxed{R = \sqrt{p^2 + \varphi^2 + 2p\varphi\cos\theta}}$$

$\Rightarrow$ expression for direction of resultant vector

from ΔOCD

$$\tan\alpha = \frac{\text{oppossite}}{\text{adjacent}}$$

$$\tan\alpha = \frac{CD}{OD}$$

here $OD = OA + AD$

$\therefore \tan\alpha = \dfrac{CD}{OA+AD}$

from ΔACD

$\sin\theta = \dfrac{CD}{AC}$

$CD = AC\sin\theta$ and $AD = AC\cos\theta$

$\tan\alpha = \dfrac{CD}{OA+AD}$

$\tan\alpha = \dfrac{AC\sin\theta}{OA+AC\cos\theta}$

here $AC = q$, $OA = p$

$\therefore \tan\alpha = \dfrac{q\sin\theta}{p+q\cos\theta}$

$$\boxed{\alpha = \tan^{-1}\dfrac{q\sin\theta}{p+q\cos\theta}}$$ //

★ 2. If the magnitude of resultant vector is equal to the magnitude of either of vector what is the angle between two vector's

Sol:- Given that $|\vec{R}| \Rightarrow R = |\vec{P}|$

(or)

$\Rightarrow R = |\vec{q}|$

$\therefore \boxed{R = p = q}$

⇒ expression for magnitude

$$R = \sqrt{p^2+q^2+2pq\cos\theta}$$

$$R = \sqrt{R^2+R^2+2\cdot R\cdot R\cos\theta}$$

$$R = \sqrt{2R^2+2R^2\cos\theta}$$

$$R = \sqrt{2R^2[1+\cos\theta]}$$

$$R = \sqrt{2R^2[2\cos^2\theta/2]}$$

$$R = \sqrt{4R^2\cos^2\theta/2}$$

$$R = \sqrt{(2R\cos\theta/2)^{\cancel{2}}}$$

$$\cancel{R} = 2\cancel{R}\cos\theta/2$$

$$0 = 2R\cos\theta$$

$$\cos\theta/2 = 1/2$$

$$\cancel{\cos}\theta/2 = \cancel{\cos}60°$$

$$\theta/2 = 60°$$

$$\boxed{\theta = 120°}$$

★★★3. If the magnitude $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then what is the angle between two vectors?

Sol: magnitude of

$$|\vec{a}+\vec{b}| = \sqrt{a^2+b^2+2ab\cos\theta}$$

$$|\vec{a}-\vec{b}| = \sqrt{a^2+b^2-2ab\cos\theta}$$

Given $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

$$\sqrt{a^2+b^2+2ab\cos\theta} = \sqrt{a^2+b^2-2ab\cos\theta}$$

$$\cancel{a^2}+\cancel{b^2}+2ab\cos\theta = \cancel{a^2}+\cancel{b^2}-2ab\cos\theta$$

$$2ab\cos\theta = -2ab\cos\theta$$

$$2ab\cos\theta+2ab\cos\theta = 0$$

$$4ab\cos\theta = 0$$

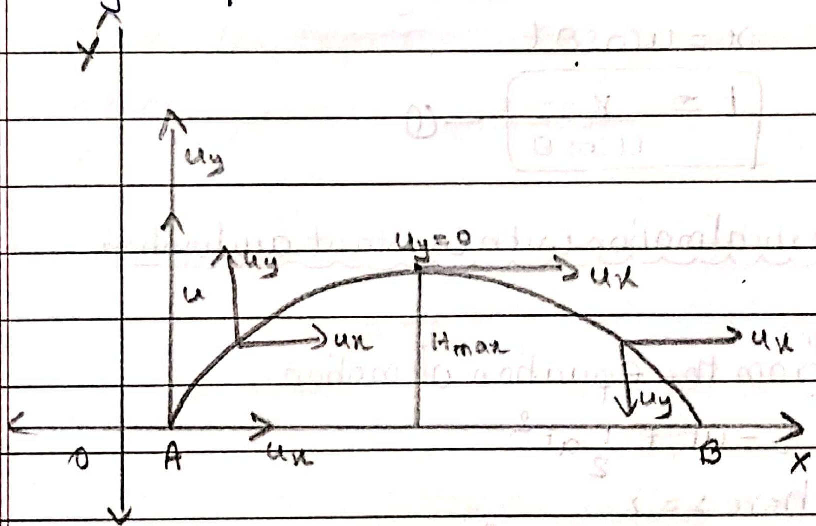
$$\cos\theta = 0$$

$$\cos\theta = \cos 90^\circ$$

$$\boxed{\theta = 90^\circ}$$

***4. Show that the trajectory thrown at a certain angle is parabola?

Sol:



→ initial velocity u can be resolved into two components

$u_x = R\cos\theta$

$u_y = R\sin\theta$

> Consider a body is thrown horizontally along x axis

> It has the initial velocity u

> Then it reaches and maximum height H_{max} and it reaches B where it touches ground

> u_x and u_y are horizontal and vertical components

⇒ horizontal motion with zero acceleration

> from the equation of motion

$s = ut + \frac{1}{2}at^2$

here $s = x$

$u = u_x = u\cos\theta$

$a = 0$ {because the body will be independent of acceleration due to gravity while moving horizontally

$\therefore x = u_x t + \frac{1}{2}(0)t^2$

$x = u_x t$

$x = u\cos\theta\, t$

$\boxed{t = \frac{x}{u\cos\theta}}$ — (1)

⇒ Vertical motion with constant acceleration

> from the equation of motion

$s = ut + \frac{1}{2}at^2$

here $s = y$

$u = u_y = u\sin\theta$

$a = -g$

$y = u_y t + \frac{1}{2}[-g]t^2$

$y = u\sin\theta\, t - \frac{g}{2}t^2$

from equation — (1)

$y = \cancel{u}\sin\theta\left(\frac{x}{\cancel{u}\cos\theta}\right) - \frac{g}{2}\left(\frac{x}{u\cos\theta}\right)^2$

$$y = \left(\frac{\sin\theta}{\cos\theta}\right) \cdot x - \frac{g}{2} \frac{x^2}{u^2\cos^2\theta}$$

$$y = (\tan\theta)x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$$

let $A = \tan\theta$

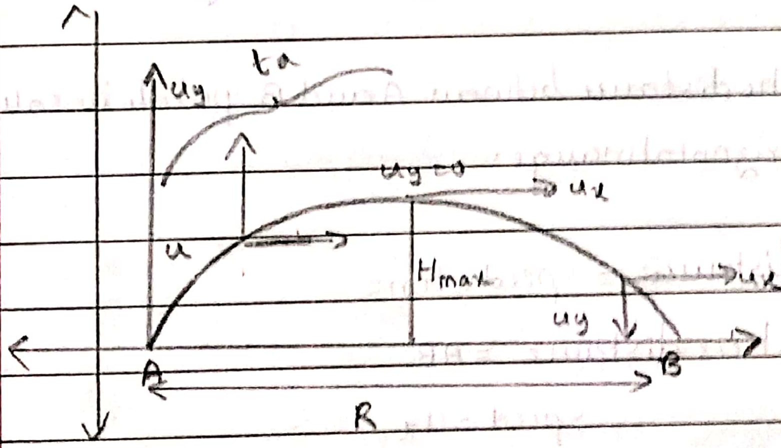
$\frac{g}{2u^2\cos^2\theta} = B$

$\therefore \boxed{y = Ax - Bx^2}$

$\therefore$ This is the equation of parabola.

★★★5. Derive the expressions for maximum height and horizontal range?

Sol:-



→ expression for maximum height

> from equation of motion

$$S = ut + \frac{1}{2}at^2$$

here $S = H_{max}$

$u = u_y = u\sin\theta$

$t = t_a = \frac{u\sin\theta}{g}$ and $a = -g$

$$\therefore H_{max} = u_y t_a + \frac{1}{2}[-g] t_a^2$$

$$H_{max} = u\sin\theta\left[\frac{u\sin\theta}{g}\right] - \frac{g}{2}\left[\frac{u\sin\theta}{g}\right]^2$$

$$H_{max} = \frac{u^2\sin^2\theta}{g} - \frac{\cancel{g}}{2}\frac{u^2\sin^2\theta}{g^{\cancel{2}}}$$

$$H_{max} = \frac{u^2\sin^2\theta}{g}\left[1-\frac{1}{2}\right]$$

$$H_{max} = \frac{u^2\sin^2\theta}{g}\left[\frac{1}{2}\right]$$

$$\boxed{H_{max} = \frac{u^2\sin^2\theta}{2g}}$$

//

→ expression for horizontal range:

> The distance between A and B points is called horizontal range.

> distance = speed x time

here distance = AB

speed = u_x

time = T $\{\because t_a + t_d\}$

$$\therefore AB = u_x \cdot T$$

> But AB = R, $u_x = u\cos\theta$, $T = \frac{2u\sin\theta}{g}$

$$\therefore R = \frac{u\cos\theta \cdot 2u\sin\theta}{g}$$

$$R = \frac{2u^2 \sin\theta \cos\theta}{g}$$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

6. What is relative velocity?

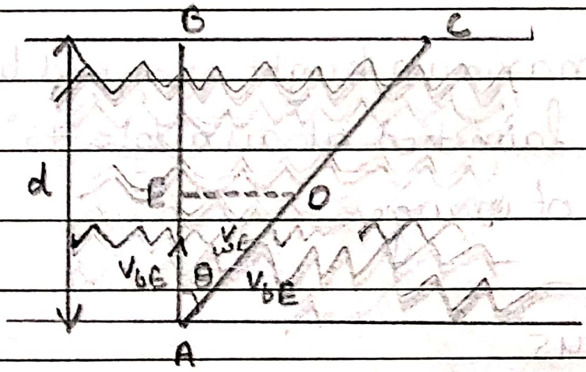
Sol: The velocity of one body with respect to another body is called as relative velocity.

If along same direction $V_{BA} = V_B - V_A$

If opposite direction $V_{BA} = V_B + V_A$

7. Show that the boat must move at an angle 90° with respect to river water in order to cross the river in minimum time?

A.



d is width of river

V_{bw} → velocity of boat with respect to water
V_{we} → velocity of water with respect to earth
V_{be} → velocity of boat with respect to earth.

from ΔADE

According to the pythagorean theorem

$$V_{BE}^2 = b_{bw}^2 + V_{we}^2$$

$$\boxed{V_{BE} = \sqrt{V_{bw}^2 + V_{WE}^2}}$$

then $\tan\theta = \frac{\text{Opposite}}{\text{adjacent}}$

$$\tan\theta = \frac{V_{WE}}{V_{BE}}$$

$$\theta = \tan^{-1}\left(\frac{V_{WE}}{V_{BE}}\right)$$

∴ The boat must move perpendicular to cross river in minimum time.

★★★ 8. Show that maximum height reached by the projectile launched at an angle 45° is one quarter of range?

Sol:

Given $\theta = 45°$

then $H_{max} = \frac{u^2 \sin^2\theta}{2g}$

$$= \frac{u^2}{2g} \sin^2 45$$

$$= \frac{u^2}{2g}\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{u^2}{2g} \cdot \frac{1}{2}$$

$$\boxed{H_{max} = u^2/4g} \quad —(1)$$

Similarly range:

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\theta = 45°$$

$$R = \frac{u^2}{g} \sin 2(45)$$

$$R = \frac{u^2}{g} \sin 90°$$

$$R = \frac{u^2}{g} \cdot 1 \quad —(2)$$

from equations (1) and (2)

$$H_{max} = \frac{u^2}{4g}$$

$$= \frac{1}{4} \cdot \frac{u^2}{g}$$

$$H_{max} = R/4$$ //

hence proved

9. Define unitvector, positionvector, nullvector?

Sol:

→ unitvector:- The vector whoose magnitude equal to 1 is Known as unit vector. It is used to specify direction

→ positionvector:- position vector is used to Specify the position of particle in space (or) used to locate the position of the particle.

→ Nullvector:- The vector whose magnitude equal to zero is called null vector.

5. Law's of motion revision

★★★ 1. What are the method's used to reduce friction?

Sol: (i) Polishing
(ii) lubricant's
(iii) ballbearing's
(iv) Streamlining.

> polishing: By polishing the surface of the bodies the frictional force between the contact surfaces decreases.
> lubricant's :- By applying the lubricants on the surface it form's a thin smooth layer between the contacts and reduces the friction.
> ballbearing's :- The wheel's of the vehicle's are provided with ball bearing's to reduce friction by replacing sliding friction with rolling friction
> Streamlining :- Automobile's and earoplane's are specially designed with curved surfaces so that the air get's streamlined during the motion hence friction is reduced.

★ 2. State law's of rolling friction?

Sol:- > Rolling friction depend's upon the area of contact
> If the area of contact is less than rolling friction is also less
> If larger is the radius of the body smaller will be the rolling friction
> Rolling friction is directly proportional to normal reaction

$$f_R \propto N$$

$$\boxed{f_R = \mu_R N}$$

★★★ 3. Explain advantages and disadvantages of friction

Sol: ⇒ Advantages:

(i) It is due to frictional force which enable's safe walking. In here the frictional force acts between foot and surface of ground

(ii) It is due to frictional force nail's and screw's are held in the wall without coming out. In here the frictional force act's between nail and surface of wall

(iii) It is due to frictional force we can hold pen in our hand.

(iv) when vehicle is moving on curved road's it help's to move without falling

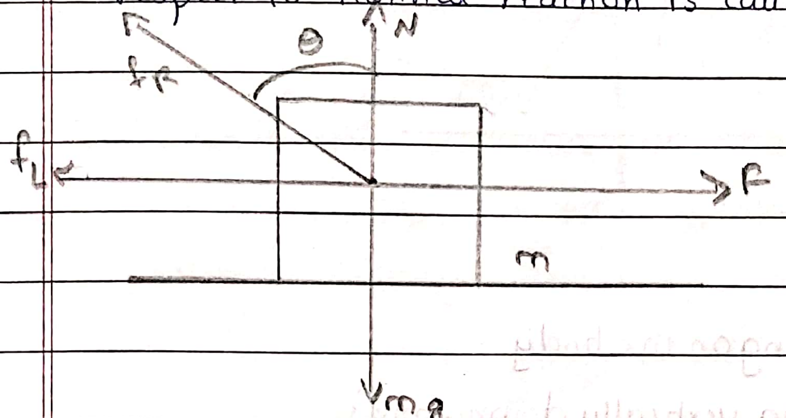
⇒ disadvantages:

(i) If there is frictional force acting on the body, we should apply more than required force to attain certain acceleration. hence it results in powerloss

(ii) By damaging the surface due to friction there will be decrease in lifetime of machines.

★★★ 4. Show that angle of friction is equal to angle of repose

Sol: ⇒ Angle of friction :- The angle made by the resultant of normal reaction and limiting friction with respect to normal reaction is called normal reaction.



here m → mass of the body
F → external force

(i) mg acting vertically downwards
(ii) N → perpendicular to plane of contact
(iii) f_L is limiting friction
where $\vec{R}$ → Resultant of {N and f_L}

OA = N = BC

OB = f_L = AC

OC = R

$$\tan\phi = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\phi = \frac{AC}{OA}$$

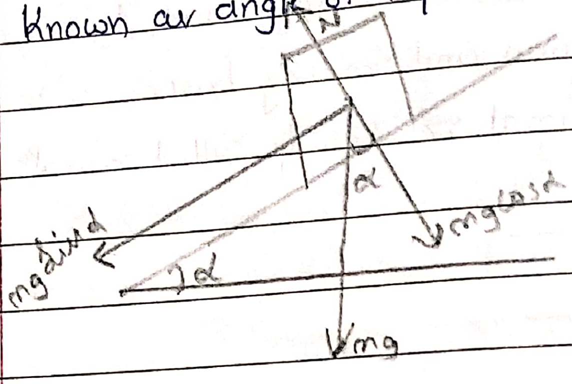
$$\tan\phi = \frac{f_L}{N}$$

$$\boxed{\frac{f_L}{N} = \mu_s}$$

$\therefore \tan\phi = \mu_s$

$\boxed{\phi = \tan^{-1}(\mu_s)}$ — (1)

⇒ angle of repose: The angle at which the body is ready to slide on an inclined plane is known as angle of repose.



⇒ forces acting on the body
(i) mg acting vertically downward's
(ii) N ⇒ perpendicular to plane of contact.
(iii) mg can be resolved into two factor's of component
→ $mg\cos\alpha$
→ $mg\sin\alpha$
f_L → limiting friction

$$N = mg\cos\alpha$$
$$f_L = mg\sin\alpha$$
$$\mu_s = \frac{f_L}{N}$$
$$\mu_s = \frac{\cancel{m}g\sin\alpha}{\cancel{m}g\cos\alpha}$$
$$\mu_s = \tan\alpha$$
$$\boxed{\alpha = \tan^{-1}(\mu_s)} \quad -(2)$$

from equation's (1) and (2)

$$\boxed{\phi = \alpha}$$

∴ angle of friction = angle of repose.

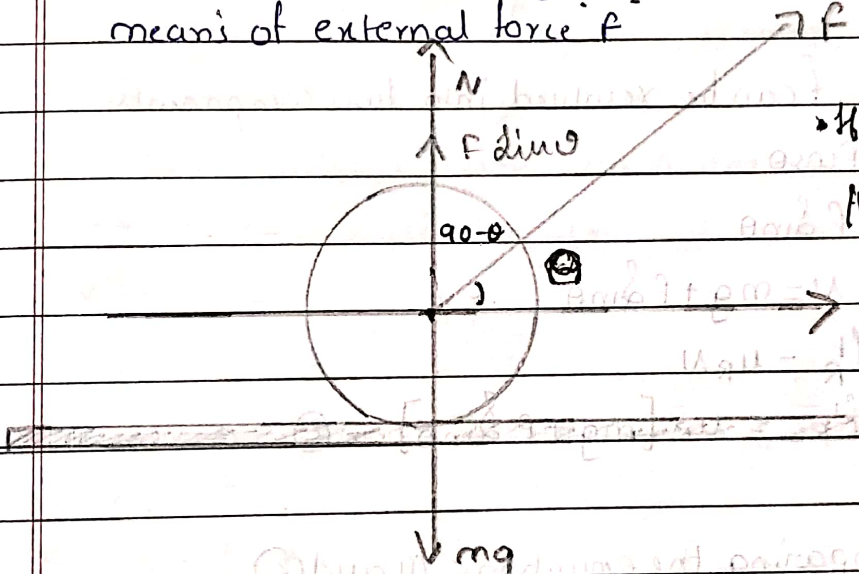
★★★ 5. Why pulling of lawn roller is prefered to pushing it ?

Sol:-

→ pulling:-

> let us consider a lawn roller of mass m

> It is rolled on a rough horizontal surface by means of external force 'F'



> If $\hat{F_1} + \hat{F_2} = \hat{F}$

$|F_1 + F_2| = \sqrt{F_1^2 + F_2^2}$

$= \sqrt{F^2 \sin^2\theta + F^2\cos^2\theta}$

$= \sqrt{F^2(1)}$

$= F$

> forces acting are

→ mg vertically downwards

→ N vertically upwards

> F could be resolved into two components

→ $F\cos\theta$

→ $F\sin\theta$

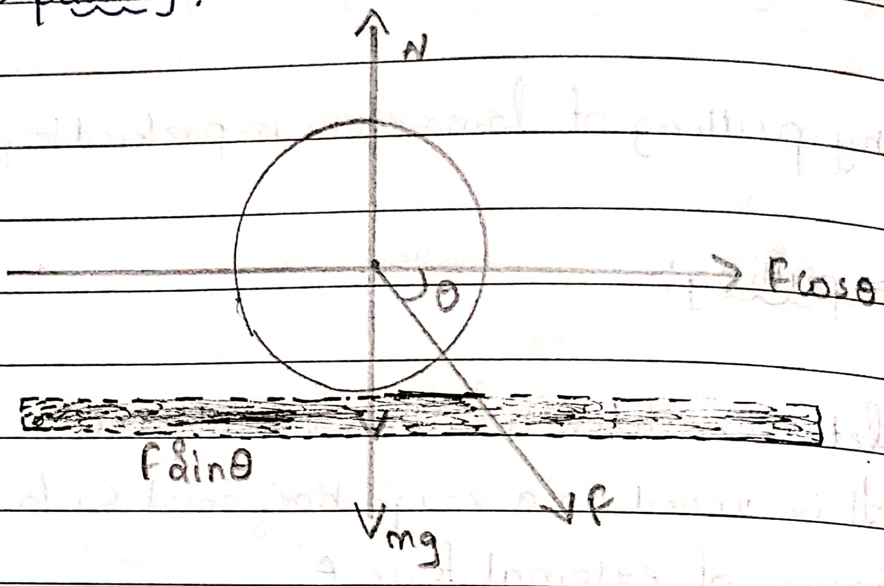
$N + F\sin\theta = mg$ {as upward forces = downward forces}

$N = mg - F\sin\theta$

as $f_R = \mu_R N$

$f_R = \mu_R [mg - F\sin\theta]$ —(1)

→ pushing :-



→ here f can be resolved into two components

→ $f\cos\theta$

→ $f\sin\theta$

$$N = mg + f\sin\theta$$

$$f'_R = \mu_R N$$

$$f'_R = \mu_R [mg + f\sin\theta] \quad - (2)$$

Comparing the equations (1) and (2)

$$f_R < f'_R$$

∴ pulling is better than pushing

II LAQ's

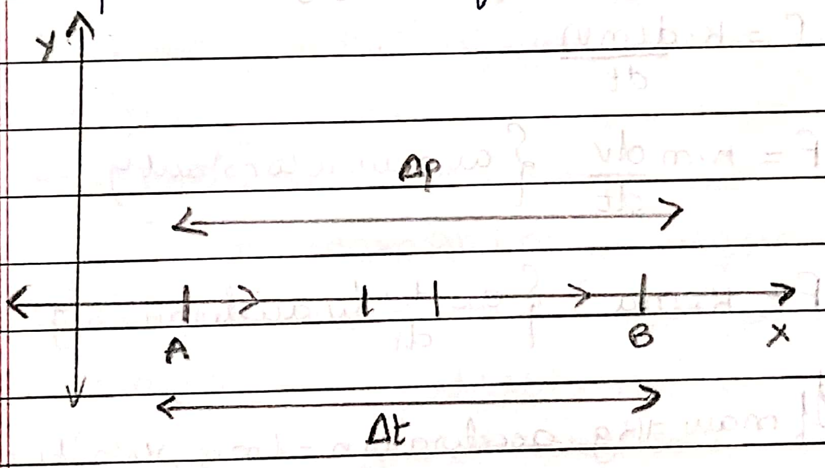
☆☆☆ 1. State newton's Second law of motion and derive $F=ma$

Sol:

⇒ Newton's second law of motion :- newton's Second law of motion states that the rate of change of momentum is directly proportional to external force acting on the body and takes place in direction in which force acts.

→ Consider a body of mass m is moving with a certain under the action of the force

→ The change in momentum of the body occurred Δp in time interval of Δt



$\Delta p \rightarrow$ change in momentum

$\Delta t \rightarrow$ time interval

$F \rightarrow$ external force

$\frac{\Delta p}{\Delta t} \rightarrow$ Rate of change of momentum

> According to newton's Second law of motion

$$F \propto \frac{\Delta p}{\Delta t}$$

> limiting the value of rate of change of momentum

$$F \propto \lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t}$$

$$\Rightarrow F \propto \frac{dp}{dt}$$

> Replacing the proportionality by using proportionality constant

$$F = K \cdot \frac{dp}{dt}$$

as $p = m \times v$

{momentum = mass × velocity}

$$F = K \cdot \frac{d(mv)}{dt}$$

$$F = K \cdot m \frac{dv}{dt}$$ {as mass is constant}

$$F = K \cdot ma$$ {as $\frac{dv}{dt}$ is acceleration a}

> If mass = 1kg, acceleration = 1 m/s, velocity = 1 m/s²

then K = 1

$\therefore \boxed{F = ma}$ hence derived

> For a body moving in circular motion centripetal force must be acting on the body.

★★ If the velocities of the objects involved in the collisions are along same str. line before and after then they are known as one dimensional ellastic collission

6. work-power-energy revesion:

I SAQ's

★★★ 1. Show that relative velocity of approach before collission is equal to relative velocity of seperation after collission?

Sol:-

→ consider two bodies of masses m_1 and m_2
→ They are travelling in a straight line
→ The initial velocities of bodies are u_1 and u_2 of bodies m_1 and m_2 respectively
→ As u_1 and $> u_2$ first body approaches second body and collides with second body
→ During collission due to exchange of energies the bodies get seperated
→ v_1 and v_2 are the final velocities of bodies m_1 and m_2 respectively.
→ The above is ellastic collission
→ hence momentum and kinetic energy remains conserved

case(i): momentum is conserved:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad -①$$

case(ii): Kinetic energy is conserved:

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_1u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\cancel{\frac{1}{2}}m_1(u_1^2 - v_1^2) = \cancel{\frac{1}{2}}m_2(v_2^2 - u_2^2)$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad -②$$

dividing the equations ②/①

$$\frac{\cancel{m_1}(u_1^2-v_1^2)}{\cancel{m_1}(u_1-v_1)} = \frac{\cancel{m_2}(v_2^2-u_2^2)}{(v_2-u_2)\cancel{m_2}}$$

$$\frac{(u_1+v_1)\cancel{(u_1-v_1)}}{\cancel{(u_1-v_1)}} = \frac{\cancel{(v_2-u_2)}(v_2+u_2)}{\cancel{(v_2-u_2)}}$$

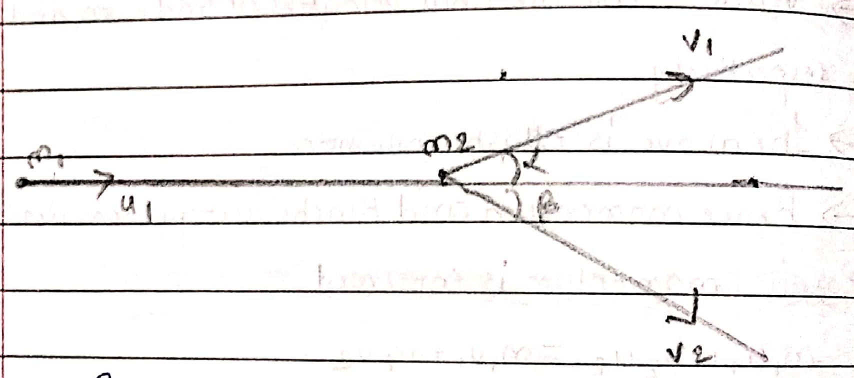
$$(u_1+v_1) = (v_2+u_2)$$

$$u_1-u_2 = v_2-v_1$$

hence relative velocity of approach = relative velocity of seperation.

★★★ 2. Show that two equal masses undergo ellastic will move right angles to each other after Collission if second body is initially at rest.

Sol:



→ Given it is ellastic Collission

→ The bodies of equal mass $m_1=m_2=m$

→ Second body is initially at rest $u_2=0$

→ As it is a ellastic collission

(i) momentum is conserved.

$$m_1\bar{u}_1 + m_2\bar{u}_2 = m_1\bar{v}_1 + m_2\bar{v}_2$$

here $m_1 m_2 = m$

$u_2 = 0$

$\therefore m_1\bar{u}_1 = m\bar{v}_1 + m\bar{v}_2$

$\cancel{m}\acute{u}_1 = \cancel{m}[\bar{v}_1 + \bar{v}_2]$

$u_1 = \bar{v}_1 + v_2$

as $R = \sqrt{p^2+p^2+2pp\cos\theta}$

$u_1 = \sqrt{v_1^2+v_2^2+2v_1v_2\cos\theta}$

$u_1^2 = v_1^2+v_2^2+2v_1v_2\cos\theta \quad -(1)$

(ii) Kinetic energy is conserved :

$\Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$\Rightarrow \frac{1}{\cancel{2}}\cancel{m}[u_1^2] + 0 = \frac{1}{\cancel{2}}\cancel{m}[v_1^2+v_2^2]$

$\therefore u_1^2 = v_1^2+v_2^2 \quad -(2)$

Substituting the value of equation -(2) in eq -(1)

$\cancel{v_1^2+v_2^2} = \cancel{v_1^2+v_2^2} + 2v_1v_2\cos\theta$

$0 = 2v_1v_1\cos\theta$

here $\theta = \alpha+\beta$

$0 = \cos(\alpha+\beta)$

$\cancel{\cos}90^\circ = \cancel{\cos}(\alpha+\beta)$

$\boxed{90^\circ = \alpha+\beta}$ // hence proved //

★★ 3. what are the differences between conservative and non conservative forces

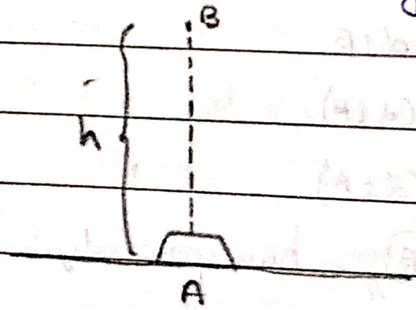
Sol:

Conservative force	non-conservative force
(i) A force is conservative if work done by the body along any closed path is zero	(i) A force is nonconservative if work done on the body by the force is not equal zero
(ii) The workdone by the conservative force doesnot depend on the path followed	(ii) The workdone by the non-conservative force depends upon the path followed.
ex:- gravitational force	eg:- frictional force.

★★ 4. What is potential energy and derive the expression for it?

Sol: potential energy: The energy possessed by the body by virtue of it's position is called potential energy

⇒ expression for potential energy:



→ Consider a body of mass m is thrown up

→ It reaches a maximum height h at point B

→ An external force F is used to lift the body in upward direction

→ To lift the body the force must be equal or greater than that of the body

$$F = mg$$

here $s = h$

$$\text{as } W = Fs\cos\theta \qquad \text{here } \theta = 0^\circ$$

$$W = Fs\cos 0^\circ$$

$$W = Fs(1)$$

$$W = mgh$$

→ here work done is in the form of potential energy

$$\boxed{U = mgh}$$

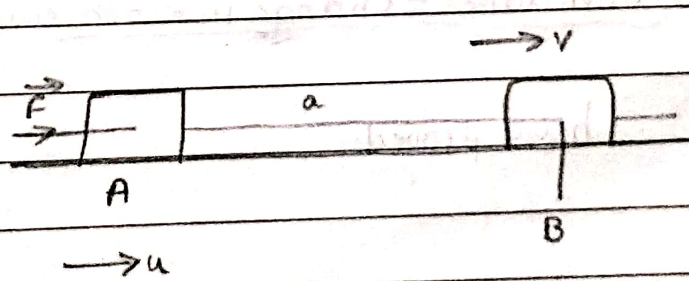
II LAQ's:-

★★★ 1. State and prove work energy theorem?

Sol:- (definition's of work and the kinetic energy and then)

work energy theorem:- The work done on the body by the force is equal to the change in kinetic energy.

→ proof:-



> Consider a body of mass m moving with initial velocity 'u'

> An external force $\vec{F}$ is acted on the body.

> Then the body will be displaced to another position with final velocity v

> It undergoes a displacement 's' with uniform acceleration 'a'

then work done $= Fs$

$$\therefore \boxed{W = Fs} \quad -(1)$$

as $F = ma$

$$\therefore \boxed{W = mas} \quad -(2)$$

from the equation of motion

$$v^2 - u^2 = 2as$$

$$\boxed{\frac{v^2 - u^2}{2s} = a} \quad -(3)$$

from the equation (2) and (3)

$$W = mas$$

$$W = m\left[\frac{v^2 - u^2}{2\cancel{s}}\right]\cancel{s}$$

$$W = m\left[\frac{v^2 - u^2}{2}\right]$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$\therefore$ work done = change in kinetic energy

hence proved.

**2. State and prove law of conservation of energy in case of a freely falling body?

Sol:-

Law conservation of energy: The total mechanical energy of the system remains if the internal forces doing work on it are conservative and external forces do no work

→ Verification of law of conservation of energy in case of a freely falling body:

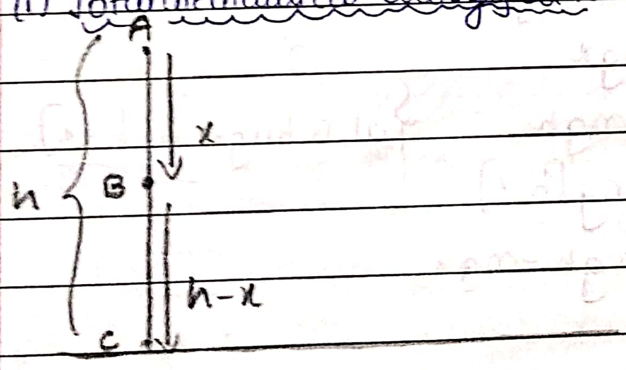
→ consider a freely falling body of mass m is dropped from certain height 'h' (at the height h, h−x and ground)

→ Consider three points in its path as A, B, C

→ E_A, E_B, E_C are the energies of A, B, C respectively

→ The total mechanical energy for each point is sum of kinetic energy and potential energy.

(i) Total mechanical energy at A



$$E_A = KE + PE$$

here $KE = \frac{1}{2}mu^2 \quad \{u=0\}$

$$K.E = 0$$

$$P.E = mgh$$

$E_A = KE + PE$

$E_A = 0 + mgh$

$\boxed{E_A = mgh}$ —(1)

(ii) Total mechanical energy at B

$E_B = KE + PE$

→ let the distance travelled by body from A to B $= x$

thus $K.E = \frac{1}{2} mv^2$

here from the equation of motion

$v^2 - u^2 = 2as$

$v^2 - 0^2 = 2gx$

$v^2 = 2gx$

$\therefore K.E = \frac{1}{2} m (2gx)$

$K.E = mgx$

thus $P.E = mgh$ {at B height is $h-x$]

$= mg[h-x]$

$= mgh - mgx$

$\therefore E_B = K.E + PE$

$= \cancel{mgx} + mgh - \cancel{mgx}$

$\boxed{E_B = mgh}$ —(2)

(iii) Total mechanical energy at c:

$E_c = KE + PE$

$KE = \frac{1}{2}mv^2$

from equation of motion $v^2 - u^2 = 2as$

$v^2 = 2gh$

$K.E = \frac{1}{2}m(2gh)$

$K.E = mgh$

$PE = mgh$ {at c height = 0}

$PE = 0$

$\therefore E_c = K.E + PE$

$E_c = mgh + 0$

$\boxed{E_c = mgh}$ — (3)

from the equation's (1), (2), (3)

$\boxed{E_A = E_B = E_C}$

" Energy is conservative under condition of gravitational force which is conservative force"

⇒ Condition's :-

(i) The total mechanical energy of the system remains constant, under the action of conservative force

(ii) The total mechanical energy of the system is not constant under the action of non conservative forces like friction and air resistance.

7. System of particles revision

I SAQ's

★★★ 1. Difference between Centre of gravity and Centre of mass

Sol:

Centre of mass	Centre of gravity
(i) Centre of mass is the point where the total mass is supposed to be concentrated	(i) Centre of gravity is the point where the total body weight acts
(ii) It refers to the mass of the body	(ii) It refers to the weight of the body.
(iii) For small and regular bodies centre of mass and centre of gravity coincide	(iii) For larger bodies centre of gravity and centre of mass do not coincide.
(iv) It does not depends on the acceleration due to gravity	(iv) It depends on the acceleration due to gravity

★★ 2. What is vector product, properties and examples?

Sol:-

⇒ Vector product (or) cross product

→ If $\vec{a}, \vec{b}$ are two vectors and θ is the angle between two vectors

then $\vec{a} \cdot \vec{b} = (ab\sin\theta)\hat{n}$

$\hat{n}$ is a unit vector used to represent the direction

→ $\hat{n}$ is perpendicular to the plane formed by $\vec{a}$ and $\vec{b}$

→ magnitude of vector product is given by

$\vec{a} \times \vec{b} = ab \sin\theta \, |\hat{n}|$

If $\vec{a} \times \vec{b} = ab \sin\theta$

∴ "vector product (or) cross product of two vector's is defined as product of magnitude of two vector's and sine angle between two vector's"

$\hat{i} \times \hat{i} = 1 \times 1 \sin 0^\circ \, \hat{n} = 0$	$\hat{i} \times \hat{j} = \hat{n}(k) = \hat{k}$
$\hat{j} \times \hat{j} = 0$	$\hat{j} \times \hat{k} = \hat{i}$
$\hat{k} \times \hat{k} = 0$	$\hat{k} \times \hat{i} = \hat{j}$

If reverse then value becomes negative.

⇒ properties of vector product:-

→ Commutative law:- If $\vec{a}$ and $\vec{b}$ are two vector's then

$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

but $\vec{a} \times \vec{b} = -[\vec{b} \times \vec{a}]$

→ distributive law:- A vector $\vec{a}$ can be distributed over a vector addition $\vec{b} + \vec{c}$ then

$\vec{a} \times [\vec{b} + \vec{c}] = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

→ If two vector's are parallel to each other

$\vec{a} \times \vec{b} = ab \sin\theta \cdot \hat{n}$

$\vec{a} \times \vec{b} = 0$

→ If two vector's are oppossite to each other

$\vec{a} \times \vec{b} = ab \sin\theta \cdot \hat{n}$

$\vec{a} \times \vec{b} = 0$

→ If two vectors are perpendicular to each other

$\vec{a} \times \vec{b} = ab \sin 90^\circ \hat{n}$

$\vec{a} \times \vec{b} = ab \hat{n}$

$|\vec{a} \times \vec{b}| = ab$

⇒ examples of vector product:

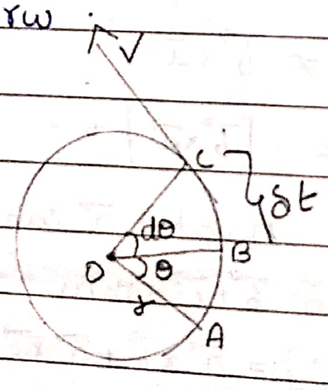
(i) If $\vec{r}$ is a position vector of any particle and $\vec{p}$ is linear momentum and $\vec{L}$ is angular momentum then

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$

(ii) If $\vec{r}$ is position vector $\vec{F}$ is the force acting on particle

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

** 3. Derive $v = r\omega$:



Sol: as $\omega = \lim_{\Delta t \to 0} \frac{\delta\theta}{\delta t}$ — ①

as Arc = radius x angle

$BC = r \times d\theta$ — ②

$V = \lim_{\Delta t \to 0} \frac{BC}{\delta t}$ — ③

from the equation's (2) and (3)

$$V = \lim_{\Delta t \to 0} \frac{r \times d\theta}{\delta t}$$

$$\Rightarrow V = \lim_{\Delta t \to 0} r \cdot \frac{d\theta}{\delta t}$$

$$\therefore V = r \cdot \lim_{\Delta t \to 0} \frac{d\theta}{\delta t}$$

$\boxed{V = r\omega}$ {from the equation - 1}

hence derived.

★★★ 4. Define angular acceleration and torque establish the relation between angular acceleration and torque.

Sol: angular accelration :- The rate of change of angular velocity is angular acceleration.

Torque : Torque (or) moment of force is defined as the product of force and perpendicular distance.

⇒ Relation between angular acceleration and torque

> τ is the torque acting on the body
> L is the angular momentum acting on the body.
> $\frac{dL}{dt}$ → Rate of change of angular momentum.

then corresponding to newton's Second law in mechanic's in rotatory motion is given by.

⇒ The rate of change of angular momentum is directly proportional to external torque acting on the body.

then torque $\propto \frac{dL}{dt}$

$$\tau \propto \frac{dL}{dt}$$

$$\tau = K \frac{dL}{dt}$$

If $K=1$

$$\boxed{\tau = \frac{dL}{dt}}$$

as we know $L = I\omega$

$$\tau = \frac{d}{dt}(I\omega)$$

$$\tau = I \cdot \frac{d\omega}{dt}$$

as $\frac{d\omega}{dt} = \alpha$ {angular momentum}

$$\boxed{\tau = I\alpha}$$

★★ 5. Explain the mass of earth moon system and it's rotation about the sun?

Sol: > The earth moon system rotates about common Centre of mass

> The mass of the earth is about 81 times that of moon. This reveals the mass of centre of earth moon system is relatively very near to the centre of system.

> The interaction of the earth and moon does not affect the motion of centre of mass of earth moon system. The gravitational attraction of sun is only external force that acts on the earth moon system and centre of mass of earth moon system moves in elliptical path around the sun.

★★★ 6. State and prove the principle of law of conservation of angular momentum?

Sol:-

Law of conservation of angular momentum :- If there is no resultant external torque on a rotating system (L), angular momentum of system remains constant in both magnitude and direction.

As $\tau = dL/dt$

If the resultant external torque is zero then

$$\frac{dL}{dt} = 0$$

$$L = \text{constant}$$

$$\text{as } L = I\omega$$

$$I\omega = \text{constant}$$

$$\therefore \quad I_1\omega_1 = I_2\omega_2$$

>examples:

(i) A man stands on a turn table with dumb-bells in his streched hands. The turn table is set into rotation at constant angular velocity ω.
If man brings his hands closer to his body angular velocity increases gradually. moves away from his body angular velocity decreases.

(ii) A ballet dancer decreases (or) increases his angular speed of rotion by streching the hands (or) bringing the hands closer to the body. hence total angular momentum remains constant.

I SAQ's.

***1. State and proove parallel axes theorem ?

Sol:

parallel axes theorem:- The moment of inertia of a rigid body about any axis is equal to the moment of inertia of a rigid body about centre of mass plus the product of mass of the body and square of perpendicular distance between two parallel axes.

$$\boxed{I = I_0 + mr^2}$$

proof:

→ Consider a rigid body of mass 'm' and contains different mass particles.

→ Consider a mass particle A passing through $\overline{AB}$ and another axes passing through centre of mass of rigid body.

→ let r be the distance between two parallel axes

then δm → mass of small particle

$$I = \Sigma \delta m (AP)^2 \quad \text{—①}$$

$$I_0 = \Sigma \delta m (CP)^2 \quad \text{—②}$$

→ then $I = \Sigma \delta m (AP)^2$ here $AP^2 = AQ^2 + PQ^2$ from traingle APQ

$\therefore AP^2 = (AC + CQ)^2 + PQ^2$ {\`as $AQ = AC + CQ$}

$$AP^2 = AC^2 + CQ^2 + 2AC\,CQ + PQ^2 \quad \text{—③}$$

from traingle CPQ

$$CP^2 = CQ^2 + PQ^2$$

Substituting value of CP^2 in eq - ③

$$AP^2 = AC^2 + CP^2 + 2AC\,CQ \quad \text{—④}$$

from equations ① and ④

$I = \Sigma \delta m [AC^2 + CQ^2 + 2AC CQ]$

$I = \Sigma \delta m AC^2 + \Sigma \delta m CQ^2 + \Sigma \delta m 2AC CQ$

$I = \Sigma \delta m AC^2 + 2\Sigma \delta m AC CQ + \Sigma \delta m CQ^2$

here $\Sigma \delta m = M$ { Sum of all mass particles $= m$ }

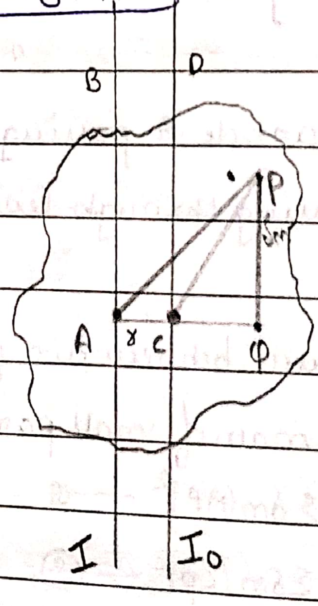
$AC = r$

$2\Sigma \delta m\, r\, CQ = 0$ { as sum of moment of forces = 0

$\Sigma \delta m CQ^2 = I_0$

$\therefore\ I = mr^2 + 0 + I_0$

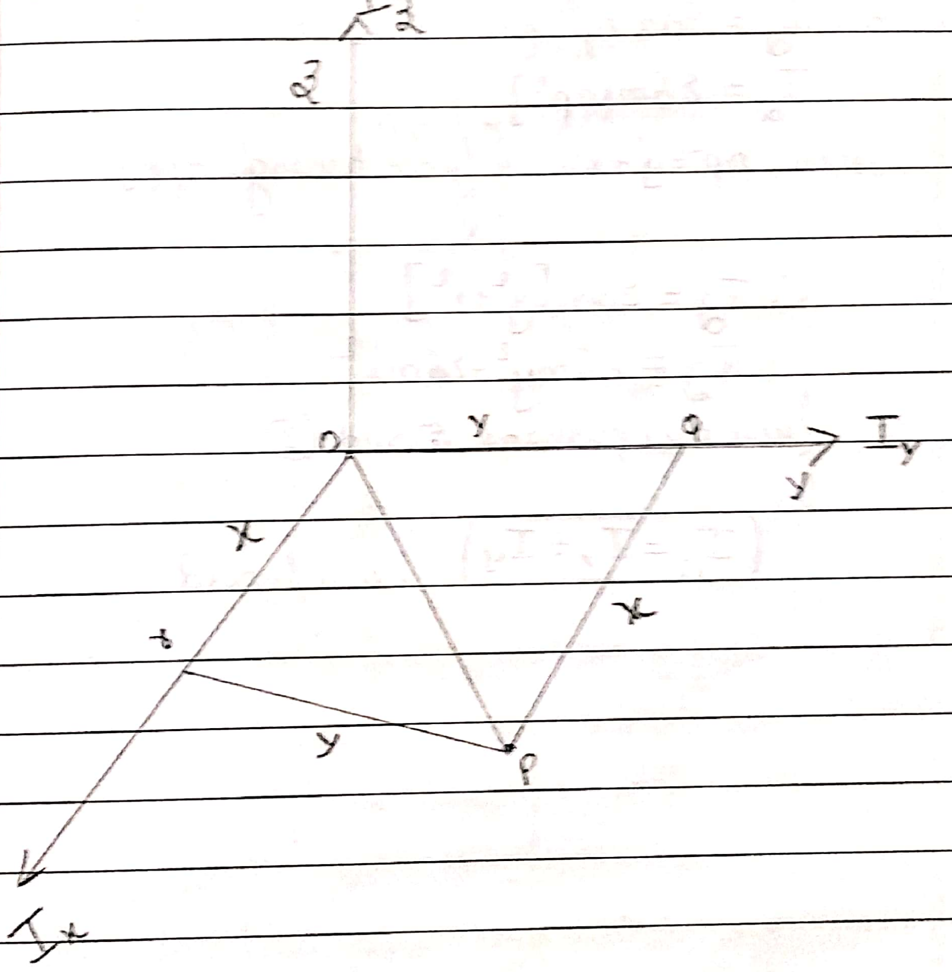
$$\boxed{I = I_0 + mr^2}$$



**2. State and proove perpendicular axes theorem !

Sol:-

perpendicular axes theorem: The moment of inertia of plane lamina about any axis is equal to the sum of moment of inertia of other two axes which are perpendicular to each other and intersecting at a point.



→ Consider a three dimensional planar lamina and a particle p in it

→ The distance between p and x axis is y

→ The distance between p and y axis is x

δm = mass of the small particle

Sum of all the mass particles $\delta m = m$

$$\Sigma \delta m = m$$

thus $I_z = \Sigma \delta m \cdot op^2$ — (1)

$I_y = \Sigma \delta m \, x^2$ — (2)

$I_x = \Sigma \delta m \cdot y^2$ — (3)

from the equation — (1)

$$I_z = \Sigma \delta m \, op^2$$

$$I_z = \Sigma \delta m [op^2]$$

here $op^2 = y^2 + x^2$ {from triangle opp}

$$\therefore I_z = \Sigma \delta m [y^2 + x^2]$$

$$I_z = \Sigma \delta m y^2 + \Sigma \delta m x^2$$

from the equations (2) and (3)

$\boxed{I_z = I_x + I_y}$ hence proved

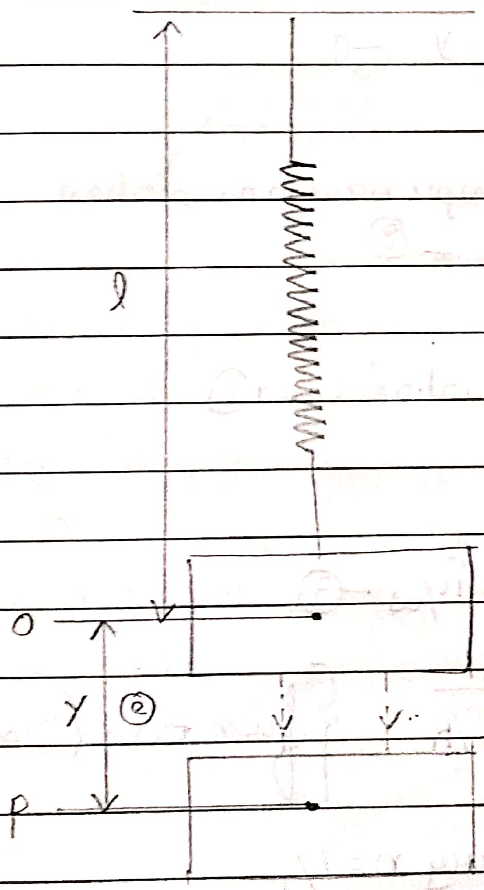
8. osscillation's revision:

I SAQ's

★★★ 1. Obtain an equation for the frequency of osscillation of spring of force constant 'K' to which mass m is attached?

Sol:

force constant (or) spring constant :- The force constant is defined as restoring force per unit extension.



→ Consider a body of mass m is attached to a spring and pulled down from point of suspension of spring.

thus $F \alpha -e$

$F \rightarrow$ restoring force

$e \rightarrow$ extension

$\therefore f = -ke$

where K is the force constant

$\therefore \boxed{K = F/e}$

units: S.I $\rightarrow$ N/m

D.F $\rightarrow [M^1L^0T^{-2}]$

as $F = -ke$

$ma = -ke$

$a = -\frac{K}{m} \cdot y$ — ①

According to simple harmonic motion

$a = (-\omega^2 x)$ — ②

$\therefore$ from the equation's ① and ②

$\omega = \sqrt{K/m}$

we know $T = 2\pi/\omega$ — ③

$\therefore T = 2\pi\sqrt{m/K}$ $\left\{ \text{from } \omega = \sqrt{K/m} \right\}$

we know frequency $n = 1/T$

$\therefore n = \frac{1}{2\pi\sqrt{m/K}}$

$\boxed{n = \frac{1}{2\pi}\sqrt{K/m}}$

//

★★ 2: Define simple harmonic motion and give two examples?

Sol:

Simple harmonic motion: A body is said to be in simple harmonic motion if it moves two and fro along a straight line about a fixed path such that at any point the acceleration is directly proportional to displacement in magnitude in oppossite direction such that aacceleration is always directed towards it's meanposition.

$$\boxed{a \propto -y}$$

where $a \rightarrow$ acceleration

$y \rightarrow$ displacement.

examples:

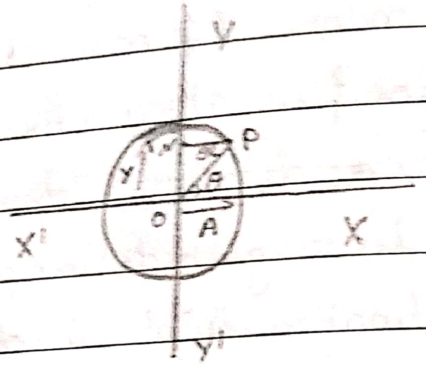
(i) oscillation's of a loaded spring suspended from a rigid support

(ii) oscillation's of a simple pendulum with small amplitude and vibration's of springs in musical instrument's

III LAQ's

***1. Show that the oscillations of projections of motion of any diameter is simple harmonic?

Sol.



where $\theta = \omega t$

→ consider a reference circle with centre 'o' and radius
→ Draw two perpendiculars from 'o' towards x axis [x and x'] and y axis as [y and y']
→ The radius from the centre of the circle is 'a'
→ The particle reaches the point p from x and reaches in t seconds with θ {angular displacement}
→ N is the foot of perpendicular from Y and y'
→ from the equation $\omega = \theta / t$

$$\boxed{\theta = \omega t} \quad -(1)$$

from the traingle opN

$$\sin\theta = \frac{opp}{hyp}$$

$$= \frac{ON}{Op}$$

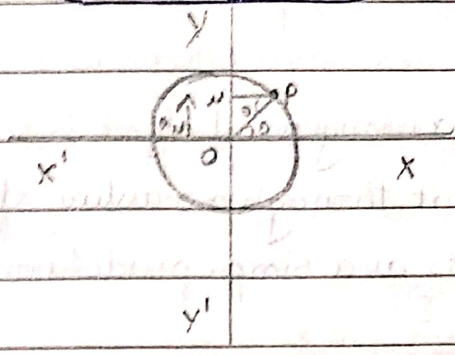
$\therefore ON = op\sin\theta$ {here $ON = Y$, $op = A$}

$$\boxed{Y = A\sin\theta}$$

from the equation —①

$\theta = \omega t$

$$\boxed{y = A \sin \omega t}$$



here $a_N \rightarrow$ acceleration of N

$a_p \rightarrow$ centripetal acceleration.

as $a_p = A\omega^2$

$\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{ON}{OP}$

$ON = OP \sin\theta$ $\left\{ \begin{array}{l} \text{here } a_p = A\omega^2 \\ \text{and } \theta = \omega t \end{array} \right\}$

$\therefore$ $a_N = a_p \sin\theta$

$a_N = A\omega^2 \sin\omega t$

$a_N = -A\omega^2 \sin\omega t$ {as towards to oppossite side}

$a_N = -\omega^2 [A \sin\omega t]$

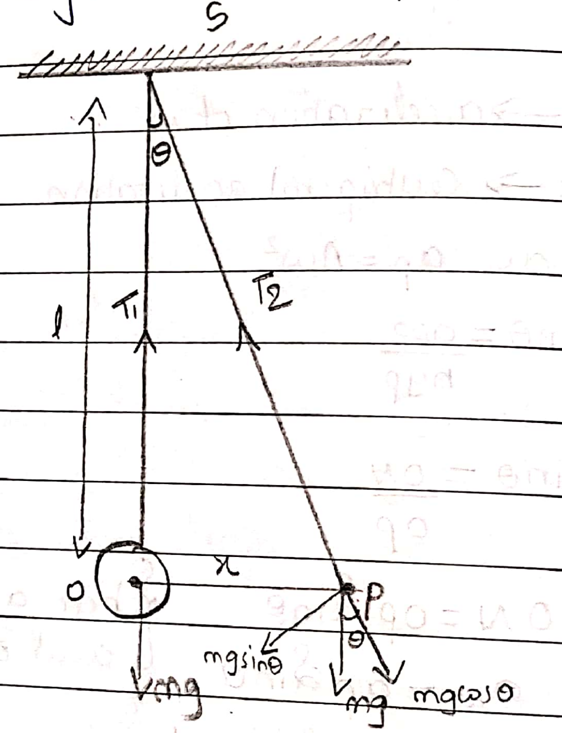
$a_N = -\omega^2 y$ {as $A \sin\omega t = y$}

$$\boxed{a_N \propto -y}$$

hence proved that acceleration of the particle is directly proportional to displacement in magnitude in appossite direction.

*** 2. what is simple pendulum? Show that the oscillations of a simple pendulum are simple harmonic and derive expresson for time period. what is seconds pendulum?

Sol: > let us consider a heavy point object is suspended from rigid support through a massless string on the string. It acts as a simple pendulum.



> In the diagram 'S' is the point of suspension and the distance between point of suspension to centre of gravity point is 'l'

> O is the equillibrium position where the bob is at the mean position. and T_1 is the Tension in the string

> As Arc = Angle × radius

$x = \theta \times l$

$\boxed{\theta = x/l}$ —①

> The body attains where the little angular displacement and reaches the point 'p'.
> At the point p forces acting on the bob are.
⇒ mg vertically downwards
⇒ T_2 acting along ps towards point of suspension.
⇒ mg can be resolved in to two compounds
> T_2 force is balanced with $mg\cos\theta$
> The unbalanced force acting on the bob is $mg\sin\theta$

$$F = mg\sin\theta$$

as $F = ma$ { here $\theta = \theta$ as angle is very minute }

$$ma = mg\sin\theta$$

$$a = g\sin\theta$$

$$\Rightarrow \boxed{a = -g\sin\theta} \quad -②$$

> The negative sign indicates the unbalanced force which is again displaced towards mean position

from the equations ① and ②

$$a = -g\{x/l\}$$ where g and l are constant

$$\boxed{a \propto -x}$$

> Therefore, the acceleration of the body is directly proportional to its displacement in magnitude in opposite direction which indicates the body is in simple harmonic motion.

⇒ expression for time period:

as $T = 2\pi\sqrt{\dfrac{\text{displacement}}{\text{time interval}}}$

$$T = 2\pi\sqrt{\frac{x}{a}}$$

$$a = g\left[\frac{x}{l}\right]$$

$$l = g\left[x/a\right]$$

$$\frac{l}{g} = \frac{x}{a}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\boxed{T \propto \sqrt{l}}$$

⇒ Seconds pendulum :- A simple pendulum whoose time period is equal to 2 seconds is Seconds pendulum

$$\boxed{T = 2 sec}$$

*** 3. Show that the osscillation's of simpile harmonic accellerator and derive the expression for kinetic energy, potential energy and total energy and energy at any point remain's constant ?

Sol:

→ expression for potential energy :- If a particle is in Simple harmonic motion then the work done by the body against the force is

workdone = Average force × displacement

here Average force $= \frac{0 + F}{2}$

$= F/2$

here displacement $= Y$

Page 57

then work done = $\frac{F}{2} \cdot y$

as $F = ma$

$w = \frac{ma}{2} \cdot y$

> If a body is in SHM then $a = \omega^2 y$

$w = \frac{m \cdot \omega^2 y}{2} \cdot y$

work done $= \frac{1}{2} m\omega^2 y^2$

> here work done is in the form of potential energy U

$\therefore \boxed{U = \frac{1}{2} m\omega^2 y^2}$ —— (1)

> Case (i) :- P.E at mean position $y = 0$

as $u = \frac{1}{2} m\omega^2 y^2$

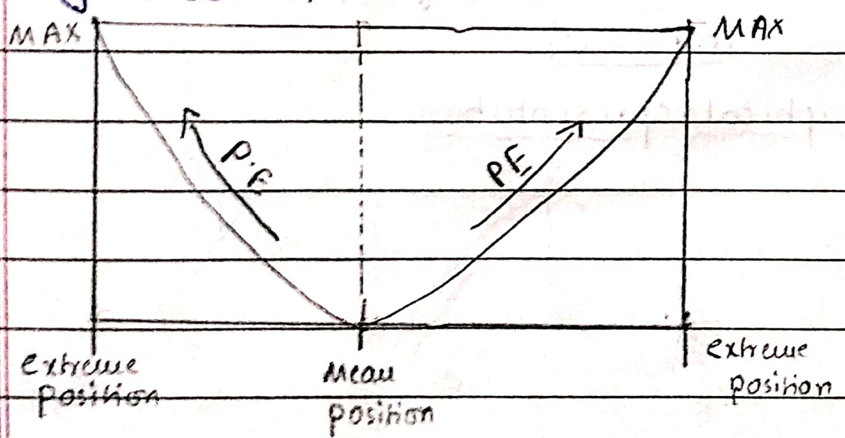
$\boxed{u = 0}$

> Case (ii) P.E at extreme position $y = A$

as $U = \frac{1}{2} m\omega^2 y^2$

$\boxed{U = \frac{1}{2} m\omega^2 A^2}$

> graphical representation :-



⇒ expression for Kinetic energy :-

> Kinetic energy $= \frac{1}{2} mv^2$

where v is velocity of osscillation

$V = \omega\sqrt{A^2 - y^2}$

> then $\boxed{\text{Kinetic energy} = \frac{1}{2} m\omega^2(A^2 - y^2)}$

> we Know as $V = A\omega\cos\omega t$

then $K.E = \frac{1}{2} mv^2$

$= \frac{1}{2} m[A\omega\cos\omega t]^2$

$\boxed{K.E = \frac{1}{2} mA^2\omega^2\cos\omega t}$ — ②

> Case(i) :- K.E at mean position $y = 0$

$\therefore K.E = \frac{1}{2} m\omega^2 [A^2 - y^2]$

$\boxed{K.E = 1/2\, m\omega^2 A^2}$

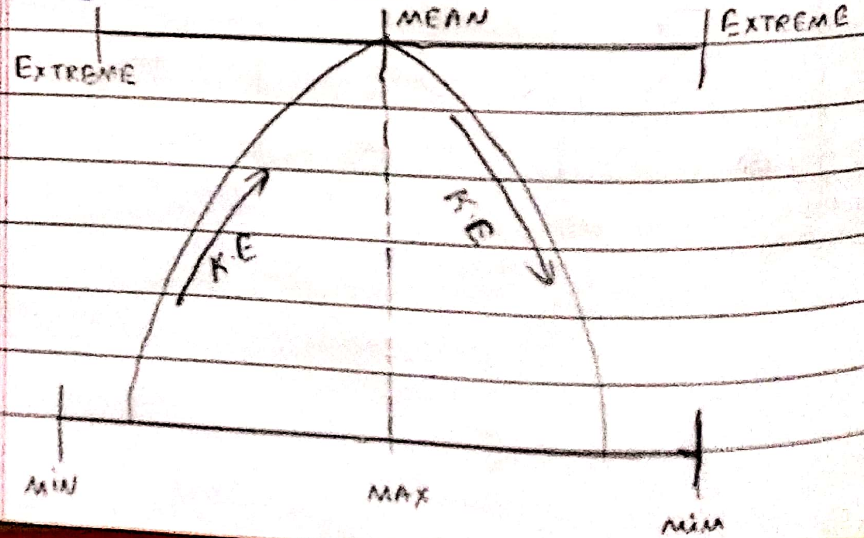
> Case(ii) :- K.E at extreme position $y = A$

as $KE = 1/2\, m\omega^2[A^2 - y^2]$

$K.E = 1/2\, m\omega^2 [A^2 - A^2]$

$\boxed{K.E = 0}$

> graphical representation :



⇒ from the case of K.E and P.E we observe that

→ $K.E_{mean} = P.E_{extreme}$

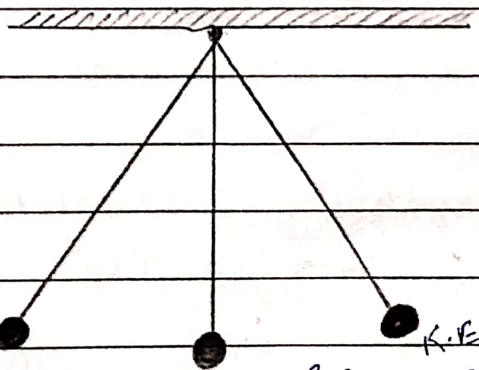
→ $K.E_{extreme} = P.E_{mean}$

⇒ expression for total energy :-

$$\text{Total energy} = K.E + P.E$$
$$= \frac{1}{2} m\omega^2 [A^2 - y^2] + \frac{1}{2} m\omega^2 [y^2]$$
$$= \frac{1}{2} m\omega^2 [A^2 - y^2 + y^2]$$
$$\text{Total energy} = \frac{1}{2} m\omega^2 A^2$$

⇒ law of conservation of energy in case of simple pendulum :-

> observing a simple pendulum considering as simple harmonic oscillator.



$K.E = 0$
$PE = \frac{1}{2} m\omega^2 A^2$
$TE = \frac{1}{2} m\omega^2 A^2$

$KE = \frac{1}{2} m\omega^2 A^2$
$PE = 0$
$TE = \frac{1}{2} m\omega^2 A^2$

$K.E = 0$
$PE = \frac{1}{2} m\omega^2 A^2$
$K.E = \frac{1}{2} m\omega^2 A^2$

⇒ hence total energy of a simple pendulum at any point is given by

$$T.E = K.E + P.E$$

$$T.E = \frac{1}{2} m\omega^2 A^2 \cos^2\omega t + \frac{1}{2} m\omega^2 A^2 \sin^2\omega t$$

$$T.E = \frac{1}{2} m\omega^2 A^2 [\cos^2\omega t + \sin^2\omega t]$$

$$T.E = \frac{1}{2} m\omega^2 A^2$$

⇒ "hence proved that the total energy at any point if the body is in simple harmonic motion remain's constant"

9. Gravitation revision :-

I SAQ :-

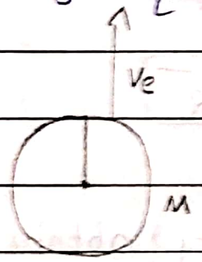
*** 1. what is escape velocity ? obtain an expression for it

Sol:- The minimum velocity required for an object to escape from gravitational influence of a planet is known as escape velocity.

⇒ expression for escape velocity :-

> let us consider a planet of mass 'm' and radius 'R'

> An object of mass 'm' is projected into the air with an velocity V_e {escape velocity}



> The gravitational potential energy of the body is given by 'mgh'

$$\therefore GPE = mgh$$
$$= m \cdot \frac{Gm}{R^2} \cdot R$$

$$GPE = \frac{GmM}{R} \quad -(1)$$

> The kinetic energy gained by the body is

$$GKE = \frac{1}{2} mv_e^2 \quad -(2)$$

⟶ According to law of conservation of energy

$$K.E = GPE$$

$$\frac{1}{2} \cancel{m} v_e^2 = \frac{G\cancel{m}M}{R}$$

$$v_e^2 = \frac{2GM}{R}$$

$$\boxed{v_e = \sqrt{\frac{2GM}{R}}}$$

→ Another expression for escape velocity
from equation

$$g = \frac{GM}{R^2}$$

$$GM = gR^2$$

$$v_e = \sqrt{\frac{2 \times g \cancel{R^2}}{\cancel{R}}}$$

$$\boxed{v_e = \sqrt{2gR}}$$

*** 2. What is orbital velocity? obtain an expression for it.

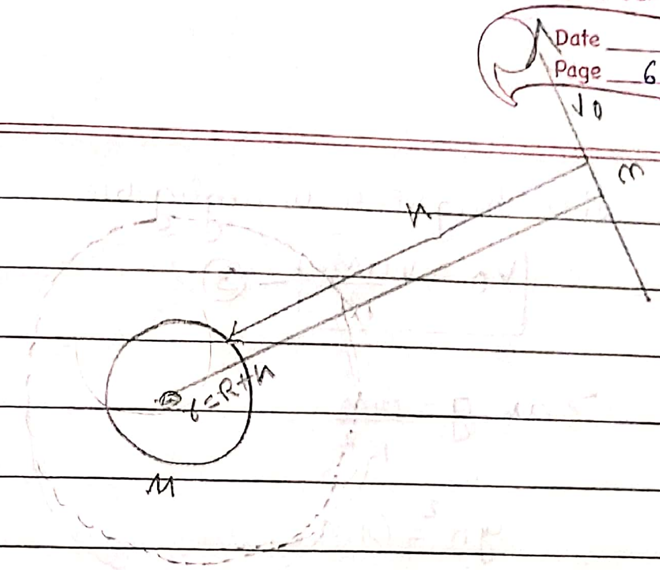
Sol:- The minimum horizontal velocity required for an object to revolve around a planet in circular orbit is called orbital velocity. It is denoted by (v_o).

⇒ expression for orbital velocity:-

> let us consider a planet of mass 'M' and radius R

> An object of mass 'm' is revolving around the planet with velocity (v_o) in circular orbits and with v_o speed.

> As it is revolving the body must experience centripetal force is given by $F = \frac{mv^2}{r}$



> The nessecary Centripetal force is provided by gravitational force

→ centripetal force is

$$F = \frac{mv^2}{r}$$

here $v = v_0$

and $r = R+h$

$$\therefore F = \frac{mv_0^2}{(R+h)} \quad —(1)$$

→ gravitational force

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{GmM}{(R+h)^2} \quad —(2)$$

from the equations (1) and (2)

$$\frac{\cancel{m}v_0^2}{\cancel{R+h}} = \frac{G\cancel{m}M}{(R+h)^{\cancel{2}}}$$

$$v_0^2 = \frac{GM}{R+h}$$

$$v_0 = \sqrt{\frac{GM}{R+h}}$$

→ If the object is placed very near to the surface of the planet then height h is negligible

$$\boxed{V_0 = \sqrt{\frac{GM}{R}}} \quad — ③$$

∵ as $g = \frac{GM}{R^2}$

$gR^2 = GM$

$V_0 = \sqrt{\frac{gR^{\cancel{2}}}{R}}$

$$\boxed{V_0 = \sqrt{gR}}$$

⇒ Relation between escape velocity and orbital velocity:

$V_e = \sqrt{2gr}$

$V_0 = \sqrt{gr}$

then $V_e = \sqrt{2} \cdot \sqrt{gr}$

$V_e = \sqrt{2} \cdot V_0$ $\quad \{as\ \sqrt{gr} = V_0\}$

$$\boxed{V_e = \sqrt{2}\, V_0}$$

***3. What is a geostationary Satellite? State it's uses!

Sol: A satellite whoose time period of revolution is equal to the period of rotation of earth {24 hours} is called geo-stationary satellites.

⇒ uses of geostationary satellite's

→ To transmitt the program's to distant places
→ To know the shape and size of earth
→ To study the upper layer's of atmosphere
→ To broadcast the changes in weather

→ To identify the minerals, resources on the surface of earth.

★★★ Q. State Kepler's law of motion?

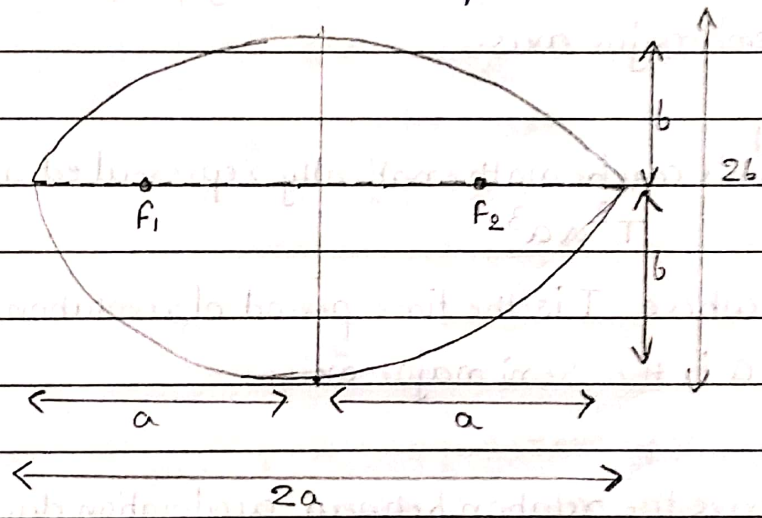
Sol:- → Kepler's law of planetary motion :-

(i) law of orbit's

(ii) law of Area's

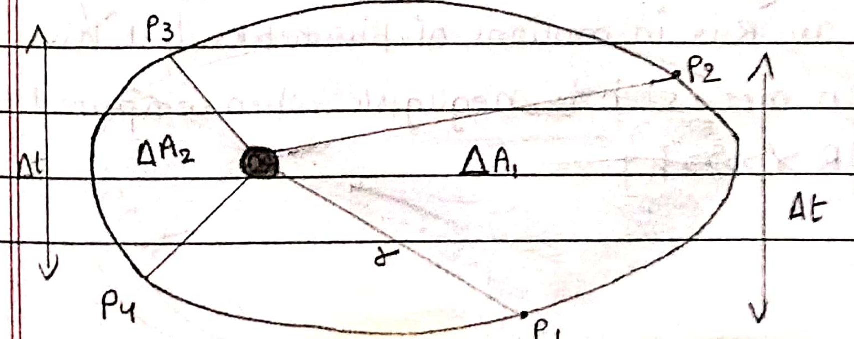
(iii) law of time periods.

⇒ Law of orbit's :- All the planets revolve around the Sun is closed circular path's called orbit's.



> Where F_1 and F_2 are the focal point's from the Centre to their semimajor and semiminor axis

⇒ Law of area's :- The line joining the planet to the Sun sweeps out the equal masses in equal intervals of time



> from the diagram

$$\boxed{\Delta A_1 = \Delta A_2}$$

here $\frac{dA}{dt}$ is constant

$\frac{dA}{dt}$ is the arieal velocity.

=> Law of time periods :- The square of time period of revolution is directly proportional to cu[be of] semi major axis.

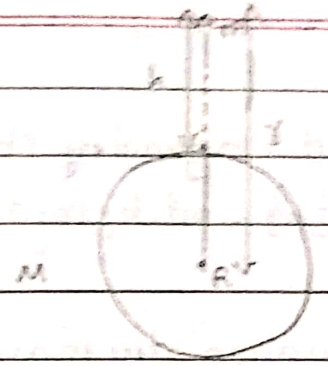
> This can be mathematically represented as

$$T^2 \propto a^3$$

> where T is the time period of revolution.
> a is the semi major axis.

** 5. Derive the relation between acceleration due to gravity (g) at the surface of planet and universal gravitational constant!

Sol:- => let us consider a planet of mass M
=> consider the radius of planet is R.
=> let us consider another body of mass 'm' at height 'h' from the surface of planet.
=> as R is in millions of kilometres but 'h' is in metres h is negligible when compared to R.
$\{R >>>>> h\}$



here $m_1 = M$ {mass of the planet}
$m_2 = m$ {mass of the object}

$r = R + h$ {as h is negligible}
$\boxed{r = R}$

> Gravitational force

$$F = \frac{G m_1 m_2}{r^2}$$

$$\therefore \quad \boxed{F = \frac{GmM}{R^2}} \quad -①$$

> gravitational pull acting on the body

$$\boxed{F = mg} \quad -②$$

from the equation's ① and ②

$$mg = \frac{GmM}{R^2}$$

$$\boxed{g = \frac{GM}{R^2}}$$

//

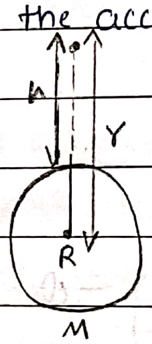
(LAQ) and (SAQ)

★★★6. how the acceleration due to gravity change for the value of height {h} and depth {d} ?

Sol:

⇒ variation of acceleration due to gravity with altitude/or height

> let us consider a planet of mass M and radius R
> An object of mass m is placed from an height 'h' from the surface of planet
> g is acceleration due to gravity on surface, and g_h is the acceleration due to gravity at 'h'



where $r = R + h$

> As the gravitational force

$$F = \frac{G m_1 m_2}{r^2}$$

here $r = R + h$

$m_1 = M$

$m_2 = m$

$$F = \frac{G M m}{(r+h)^2} \quad \text{—(1)}$$

> gravitational pull acting on the body.

$$F = m g_h \quad \text{—(2)}$$

from the equations ① and ②

$$\cancel{m} g_h = \frac{GM\cancel{m}}{(r+h)^2}$$

$$g_h = \frac{GM}{(r+h)^2}$$

here from the relation.

$$g = \frac{GM}{R^2}$$

$$GM = gR^2$$

$$g_h = \frac{gR^2}{(r+h)^2}$$

$$g_h = \frac{g\cancel{R^2}}{\cancel{R^2}\left[1+\frac{h}{R}\right]^2}$$

$$g_h = \frac{g}{\left[1+\frac{h}{R}\right]^2}$$

$$\boxed{g_h = g\left[1+\frac{h}{R}\right]^{-2}}$$

* It is the formula used for shorter distances of height as in 1000 (or) 2000 m
* The formula used for longer distances of height is

binomial expression :-

$(1+x)^n = 1+nx$ where, $x <<< 1$

$$g_h = g\left[1+(-2)\frac{h}{R}\right]$$

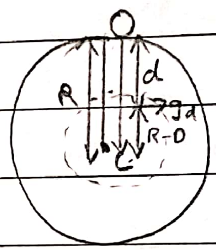
$$\boxed{g_h = g\left[1-\frac{2h}{R}\right]}$$

⇒ Variation of acceleration due to gravity:

> let us consider a planet of mass 'M' and radius of 'R'

> An object of mass {small} m is placed at certain depth from the surface of planet.

> g_d is the acceleration due to gravity at depth 'd'



> Consider a body is initially at A then is displaced to 'B'

as $g = \frac{GM}{R^2}$

here m → mass of the body

as mass is product of volume and density

$m = V \times \rho$

$m = \frac{4}{3}\pi r^3 \rho$ $\{r = R\}$

$\therefore \boxed{g = G\frac{4}{3}\pi R\rho}$ —①

and $g_d = \frac{4}{3}\pi(R-d)\rho G$ —②

> dividing the equation's $\frac{②}{①}$

$$\frac{g_d}{g} = \frac{\cancel{4/3}\,\cancel{\pi}\,\cancel{G}(R-d)\cancel{\rho}}{\cancel{4/3}\,\cancel{\pi}\,\cancel{G}R\cancel{\rho}}$$

$$\frac{g_d}{g} = \frac{R-d}{R}$$

$$\frac{g_d}{g} = \frac{R}{R} - \frac{d}{R}$$

$$\frac{g_d}{g} = 1 - \frac{d}{R}$$

$$\boxed{g_d = g\left[1-\frac{d}{R}\right]}$$

//

II. LAQ's:

** 1. Define gravitational potential energy and derive an expression for it using of association with m_1 and m_2?

Sol:

⇒ Gravitational potential energy:- The amount of work done to bring an object from infinity to a point in the feild is called gravitational potential energy.

⇒ explanation:-

> Consider a body of maw 'm' at some infinity point

> Consider another body of maw 'M' at point A

> As same work is done the body moves to point p with dx displacement.

> Then it reaches another point q to then point 'B'

> Then the distance from A to B is x

> The total distance travelled from ∝ to A point (x distance) is x

> Then the work done is

$$\boxed{dw = F \cdot dx} \quad -①$$

as $F = \frac{G m_1 m_2}{R^2}$

$$F = \frac{GmM}{x^2} \quad -②$$

$$\therefore dw = \frac{GmM}{x^2} \cdot dx$$

$$\int d\cancel{w} = \int_{\infty}^{r} \frac{GmM}{x^2} \cdot dx$$

→ As if there is no function the integration will be antiderivative.

$$w = \int_{\infty}^{r} \frac{GmM}{x^2} dx$$

$$w = GmM \int_{\infty}^{r} x^{-2} dx$$

as $\int x^n dx = \frac{x^{n+1}}{n+1}$

$$\therefore w = GmM \left[\frac{x^{-2+1}}{-2+1} \right]_{\infty}^{r}$$

$$w = GmM \left[\frac{x^{-1}}{-1} \right]_{\infty}^{r}$$

$$w = -GmM \left[\frac{1}{x} \right]_{\infty}^{r}$$

$$\omega = -GmM\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\omega = -GmM\left[\frac{1}{r} - 0\right]$$

$$\omega = \frac{-GmM}{r}$$

or $\omega = GPE$

$$\boxed{GPE = -\frac{GmM}{r}}$$

★ 2. State newton's law of gravitation and determine the value of universal gravitation constant using Cavendish method?

Sol:-

⇒ Newton's law of universal gravitation :- Every particle in the universe attracts every other particle with some force and is directly proportional to the product of masses and inversly proportional to square of distance between two bodies.

$$\boxed{F = \frac{Gm_1m_2}{r^2}}$$

⇒ Determination of universal gravitational constant using cavendish method :-

⇒ Consider a rod is suspended from a rigid support through a massless spring.

→ Two free end points of rod of length 'l' are 'A' and 'B'

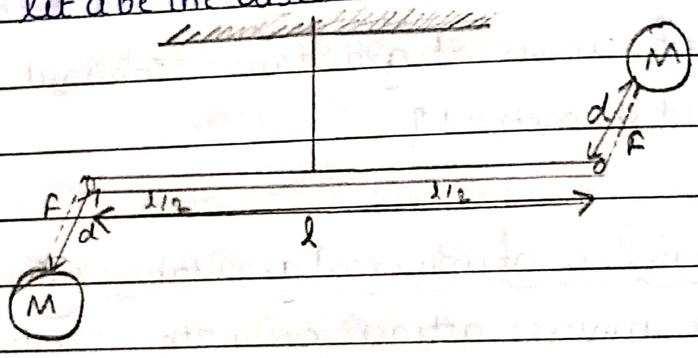
→ Two equal masses are attached to each of end point.

→ Consider two bodies of spherical shape of equal mass M diagonally one for A point in downward direction.

→ Another body is placed on upward direction of 'B'

→ According to newton's law of universal gravitation the force acting between two bodies

→ let d be the distance between two bodies



> as torque = force x perpendicular distance

$$T_1 = F \times \frac{l}{2} + F \times l/2$$

$$T_1 = \cancel{2} F \times \frac{l}{\cancel{2}}$$

$$\boxed{T_1 = Fl} \quad \text{—} (1)$$

> here F ⇒ gravitational force acting

$$F = \frac{GmM}{r^2}$$

$$F = \frac{GmM}{d^2} \quad \text{—} (2)$$

> from the equation's (1) and (2)

$$T_1 = \frac{GmM}{d^2} \cdot l \quad \text{—} (3)$$

> As two forces acting on the rod it deflects from its original position with angle θ.

$\therefore$ The restoring torque is

$$T_2 = K\theta \quad \text{--- (4)}$$

as the forces are equal

$$T_1 = T_2$$

$$l \cdot \frac{GmM}{d^2} = K\theta$$

$$l \cdot G = \frac{K\theta d^2}{mM}$$

$$G = \frac{K\theta d^2}{lmM}$$

By substituting the values experimentally we get

$$\boxed{G = 6.6 \times 10^{-11}\ Nm^2 kg^{-2}}$$ //

10. Mechanical properties of solid's revesion

I SAQ's

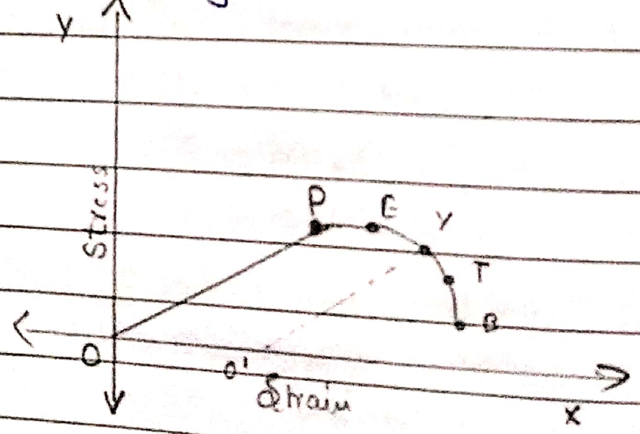
***1. Describe the behaviour of wire on gradually increased load?

Sol: → Consider a wire that is suspended from a rigid support.

→ Attach a load of mass to the free end of the wire which acts as deformation force

→ Due to the deformation force elongation in the wire takes place and elongation continues to a limit called as proportionality limit

→ proportionality limit {P} :- The maximum stress applied on the wire upto which stress is directly proportional to strain. obeying hooke's law is called "proportionality limit". OA is the straight line. If the wire is released it regains back it's original shape and size



⇒ If the stress is further increased from P then the wire does not obey hookes law. But still the ellastic nature of wire exists hence maximum stress produced to which without effecting the "ellastic limit". point P and E coincide for some materials PE is the slight curve elongation

⇒ Yeilding point :- If the stress crosses the ellastic limit then the wire become's streched permanently EY in the elongation.

⇒ Tensile point :- After crossing the Yeilding point the wire continues to elongate even if the stress is not increased as wire becomes thin at 'T'

⇒ Breaking point :- After T point the wire begin's to become thin and finally it breakes at B. The stress at Breaking point is breaking stress

★★★ 2. Explain the concept of ellastic potential energy for a streched wire and derive an expression for it ?

Sol:

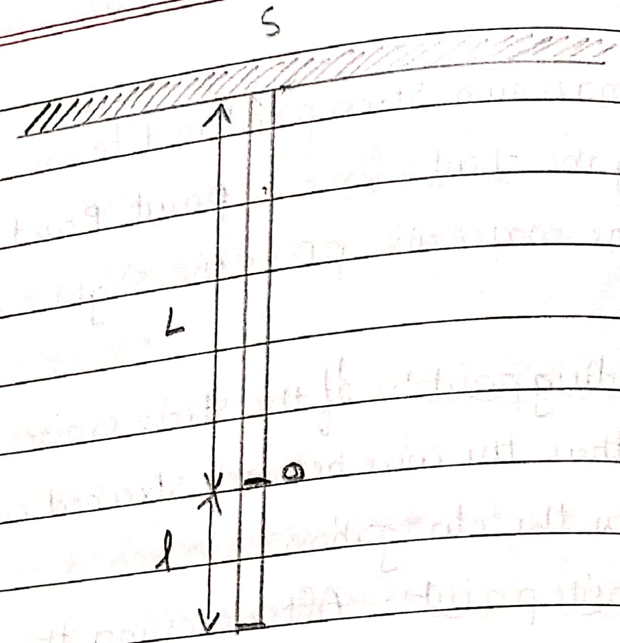
⇒ ellastic potential energy :- The work done in streching a wire is called ellastic potential energy.

⇒ expression for ellastic potential energy :-

→ Consider a wire of length 'L' suspended from a rigid support

→ By applying the deformation force the elongation in wire takes place (e) or (l) from equillibrium point.

→ F is the force acting on the free end of wire



> Y is the Young's modulus

$$Y = \frac{F \cdot L}{A \cdot l}$$

$$\boxed{F = \frac{Y \cdot A l}{L}} \quad — ①$$

> here work done is

$$dw = F \cdot dl$$

> Take integration on both sides

$$\int dw = \int d \cdot F \cdot l$$

> If there is no function then the integration will be antiderivative

$$w = \int F dl \quad — ②$$

from the equations ① and ②

$$w = \int \frac{YAl}{L} \cdot dl$$

$$w = \frac{YA}{L} \int l' dl$$

as $\boxed{\int x^n dx = \frac{x^{n+1}}{n+1}}$ {Integration formula}

$$\omega = \frac{YA}{L}\left[\frac{l^{1+1}}{1+1}\right]$$

$$\omega = \frac{YA}{L}\left[\frac{l^2}{2}\right]$$

$$\omega = \frac{YAl^2}{2L}$$

$$\omega = \frac{1}{2}\left(\frac{YAl}{L}\right)l \quad \{\text{from} - (1)\}$$

$$\omega = \frac{1}{2} F . l$$

$$\omega = \frac{1}{2} . F . l$$

> here work done is in the form of potential energy

$$U = \frac{1}{2} F . l$$

$$U = \frac{1}{2} \times \text{force} \times \text{extension in wire.}$$

** 3. Define modulus of stress, strain and poission's ratio?

Sol:- Modulus of ellasticity :- within the proportionality limit, the ratio of stress to the corresponding strain of body is called modulus of ellasticity.

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Strain :- The ratio of change of dimension to the original dimension of a body is called strain.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Stress:- The internal restoring force acting per unit area, setup inside a body is called stress

$$\text{Stress} = \frac{\text{force}}{\text{area}}$$

$\therefore$ Stress = F/A

poisson's ratio:- poission's ratio is the ratio of lateral strain to longitudinal strain

$$\therefore \text{poisson's ratio} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\sigma = \frac{-dD / D}{e/L}$$

4. explain stress and kind's of stress?

Sol. Stress:- The restoring force per unit area is setup inside the body called as stress

$$\text{Stress} = \frac{\text{Restoring force}}{\text{unit area}}$$

$$= \frac{\text{deformation force}}{\text{unit area}}$$

$\boxed{\text{Stress} = \frac{F}{\pi R^2}}$ as $A = \pi r^2$

$\Rightarrow$ Types of stress are:

(i) Normal stress: If the stress is normal to the body (or) Surface of the body then it is caused as normal

Stress.

→ normal stress are of two types:-

(i) longitudinal stress

(ii) volume stress (or) bulk stress

→ longitudinal stress: when the normal stress changes the length of the body then it is called as longitudinal stress.

$$\text{longitudinal stress} = \frac{\text{deformation force}}{\text{unit area}}$$

$$= F/A$$

→ volume stress (or) bulk stress: when the normal stress changes the volume of the body then it's called as volume stress (or) bulk stress.

$$\text{Volume stress} = \frac{\text{Force}}{\text{Area}}$$

$$\Rightarrow F/A = P$$

(iii) Tangential (or) Shearing stress: when the stress is tangential to the surface due to application of parallel forces to the surface then it is called as Tangential stress (or) Shearing stress

$$\text{Tangential stress} = \frac{\text{Force}}{\text{Surface area}} = F/A$$

** 5. explain strain and kinds of strain ?

Sd.

⇒ longitudinal strain :- longitudinal strain is the ratio of change in length to original length.

$$\text{longitudinal strain} = \frac{\text{change in length}}{\text{Original length}}$$

$$= e/L$$

⇒ volume stress (or) bulk stress :- It is the change in volume to the original volume.

$$\text{Volume strain} = \frac{\text{change in volume}}{\text{Original volume}}$$

If Δv is the change in volume and v is the Original volume then :

$$\text{Volume strain} = \frac{\Delta V}{V}$$

⇒ Tangential strain (or) Shearing strain : It is the angle through which a face is turned

$\theta \to$ Tangential strain

Arc = Angle x radius

$l = \theta \times L$

$$\boxed{\theta = \frac{l}{L}}$$

Q II LAQ's

**1. Define hookes law and determine the young's modulus of a given wire?

Sol: hookes law: hookes law states that stress is directly proportional to corresponding strain in respect with proportionality limit.

Stress & Strain

=> Determination of young's modulus of a wire:

> let us consider two wires A and B of same length and same material and area

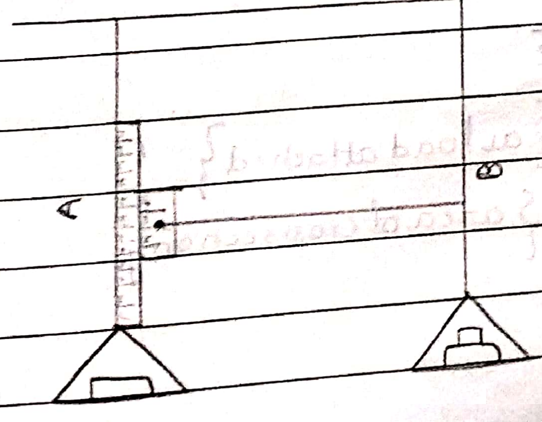
> A wire can be considered as reference wire and 'B' can be considered as experimental wire.

> Theese two wires are suspended from rigid support.

> Two equal weighing pans are attached to reference and experimental free wire end's.

> Mainscale is attached to reference wire and noted as 'M'

> Vernier scale is attached to B on showing the pointer in respect with the vernier scale 'V'



⇒ working:

> The original length of wire L can be measured by using scale and screw guage initially

> As to make the wire stiff and straight. we add two equal certain mass of weights to weighing pans

> let take reading on the vernier and note it as R_1

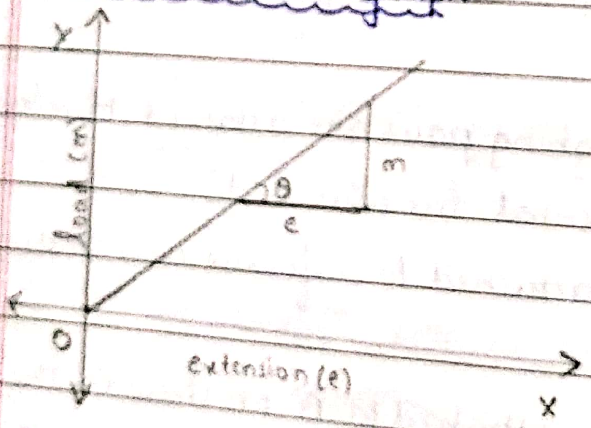
> Further add load to weighing pan of (B) experiment wire so elongation produces.

> Then vernier reading is noted down as R_2

> Thus extension in wire is

$$\boxed{e = R_2 - R_1}$$

⇒ load-extension graph



> According to Youngs modulus

$$Y = \frac{F \cdot L}{A \cdot e}$$

as $F = mg$ {as load attached}

$A = \pi r^2$ {area of cross section}

$$Y = \frac{mg \cdot L}{\pi r^2 \cdot e}$$

$$Y = \frac{m \, gL}{e \cdot \pi r^2}$$

$$Y = \frac{M}{e} \cdot \frac{gL}{\pi r^2}$$

⇒ Conclusion :- By substituting M/e value in the above expression we get Young's modulus of given wire.

11. mechanical properties of fluid's revision:

I SAQ's

★★★1. Explain surface tension and surface energy?

Sol:

→ Surface tension:- Surface tension is defined as the tangential force per unit length at right angle's on either side of line, imagine to be drawn on the free surface of liquid in equilibrium.

The free surface of the liquid behaves as a streched rubber sheet.

Surface tension → T

Force → F

length → l

∴ Surface tension = $\frac{\text{force}}{\text{unit length}}$

$T = F/l$

> units are N/m

> dimensional formula = $[M^1L^0T^{-2}]$

→ Surface energy:- The additional potential energy due to molecular forces per unit surface area is called surface energy classmate

Date

Page 87

→ The free surface of the liquid contains molecules and this molecule experiences a downward force due to which a molecule possesses a additional potential energy.

$$\text{Surface energy} = \frac{\text{additional potential energy}}{\text{Surface area}}$$

$$S = \frac{W}{A}$$

> units are S.I → J/m^2

★★★ 2. explain dynamic lift with examples !

Sol: → Dynamic lift :- dynamic lift is the force acting on the body due to pressure difference

⇒ examples :-

(i) dynamic lift acting on the aeroplane (or) aircraft wing :

→ Consider a aircraft wing having concave structure at its bottom.

→ V_T is the velocity of the fluid at the top of the aircraft wing

→ V_B is the velocity of the fluid at bottom of the aircraft wing.

→ Velocity at top of the aircraft wing is greater than velocity at bottom due to pressure difference

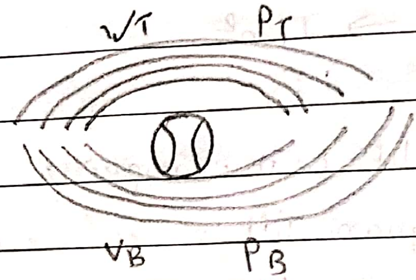
$P_B > P_T$

→ As the aircraft moves the air gets streamlined and aircraft moves in upward direction.

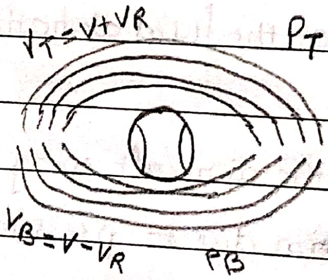
→ In this condition dynamic lift acts on the aircraft wing.

(ii) dynamic lift on a spinning ball

→ Consider a ball in a fluid without spinning
→ V_T and V_B are the velocities of the ball at top and bottom of the fluid
→ P_T and P_B are the velocities of fluid at top and bottom



→ here $V_T = V_B$ and $P_T = P_B$ Therefore no pressure difference
→ Consider another ball in a fluid spinning
→ v is the original velocity of the fluid.
→ V_r is the velocity obtained due to rotation of the ball.
→ $V_T = V + V_r$
$V_B = V - V_R$



here $V_B > V_T$
$P_B > P_T$

→ Therefore due to pressure difference dynamic lift acting on the body.

***3. what is toricellis law? explain how speed of efflux can be determined by experiment?

Sol:

Torricellis law:- The speed of efflux from an offrice at the side of the container at depth h below the free surface of the liquid is equal to the speed gained by a freely falling body after when it falls through a distance h

experiment:-

→ Consider a tank containing a lipuid of density ρ with small hole in it's side at height y_1 from bottom.

→ let the free surface of the liquid be at height y_2 from bottom. let P_a be the atmospheric pressure.

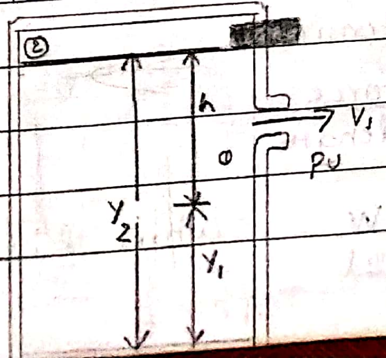
→ By applying bernoulis theorem and neglecting gause pressure

$$\cancel{P_a} + \frac{1}{2}\rho v_1^2 + \rho g y_1 = \cancel{P_a} + \rho g y_2$$

$$\Rightarrow \frac{1}{2}\rho v_1^2 = \rho g(y_2 - y_1)$$

→ Assuming $y_2 - y_1 = h$, $h = \frac{v^2}{2g} \Rightarrow v = \sqrt{2gh}$

→ This is the speed of freely falling body at any point h during it's fall. The above equation is called toricellis law



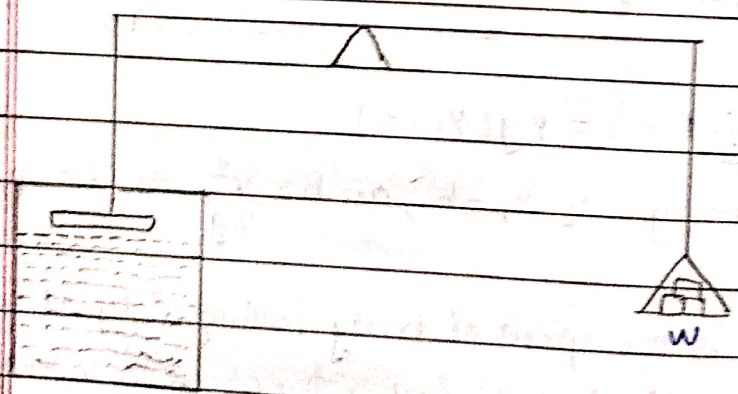
***4. explain how surface tension can be determined experimentally

Sol: → Consider a flat vertical glass plate, below the glass plate is suspended from one of the arm's of a balance

→ The other arm contain's a weighing pan with weight's

→ let the plate is balanced on the other side with it's horizontal edge just over the water. The vessel is raised slightly till the liquid just touches the glass plate and the glass plate is pulled little down due to the surface tension. Again the weight's are added till the plate just leaves the Contact with water

→ The length of glass rod is 2l, as both the surfaces are in contact with water



→ As surface tension

$$T = \frac{Force}{length}$$

$$T = \frac{W}{2l}$$ where l is length of plate

→ By substituting the values we get surface tension.

***5. Explain hydraulic lift and hydraulic brakes:

Sol: ⇒ hydraulic lift :- The working of hydraulic lift is based on the pascal's law.

→ In a hydraulic lift, there are two piston's with different area's of cross section A_1 and A_2 of P_1 and P_2 respectively.

→ A piston of small area of cross section is directly used to Apply F_1 force on the liquid

$$P = \frac{F_1}{A_1}$$

→ This pressure is transferred to the another piston which result's in an upward force F_2 in second piston.

$$\therefore P_1 = F_1/A_1$$

$$P_2 = F_2/A_2$$

→ According to pascal's law

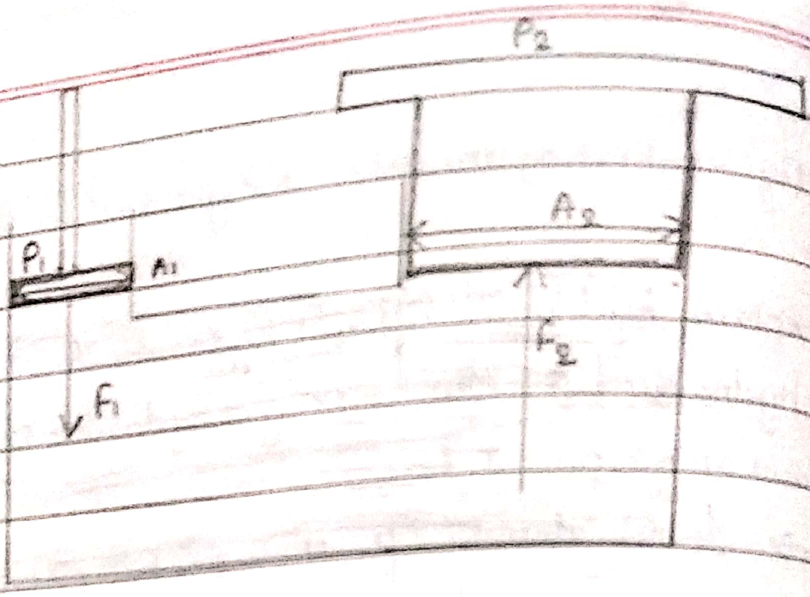
$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \frac{F_1 A_2}{A_1}$$

$$\boxed{F_2 \propto \frac{1}{A_1}}$$

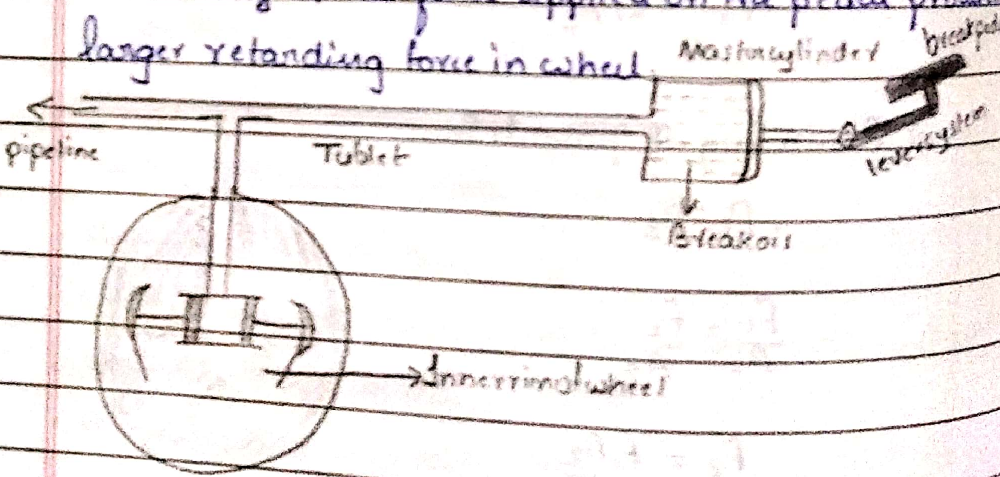
→ If the area of cross section is less more force act's on the second piston



⇒ hydraulic brakes:

→ In automobiles, hydraulic brakes work on the principle of pascal's law. when littte force is applied on the break pedal, the master piston moves inside the master cylinder and the pressure is transmitted through the brake oil to act upon the piston of larger area.

→ A large force acts on the piston and is pushed down expanding the brakes shoes against the break lining. In this way small force applied on the pedal produces larger retarding force in wheel.



**6. what is reynold's number what is it's significance?

Sol: Reynold's number :- Reynold's number is a dimensionless number, whoose value gives approximate idea wether the flow is steady (or) turbulent, then it is called as Reynold number.

→ when the rate of fluid is small the flow will be steady, when the rate of fluid flow is large the flow becomes turbulant. In turbulant flow, the velocity of fluid's at any point in space varies rapidly and randomly with time.

$$\therefore \quad R_e = \frac{\rho v d}{\eta}$$

→ where ρ is the density of fluid flowing with a speed v, and d is the dimension {length or diameter} of pipe, η is the coefficient of viscocity of liquid.

→ It is found that the flow is streamlined for $R_e < 1000$. If $R_e > 2000$ then it is turbulant fluid flow.

* 7. What is atmospheric pressure and how it is determined using barometre?

Sol: → The pressure of atmosphere at any point is equal to the weight of the coloumn of air of unit crosssectional area extending from that point to top of atmosphere.

→ At sealevel atmospheric pressure is 1.013×10^5 Pa (1 atm). Atmospheric pressure is measured using a device known as barometre.

→ mercury barometre consists of long glass tube closed at one end filled with mercury inverted into a trough of mercury

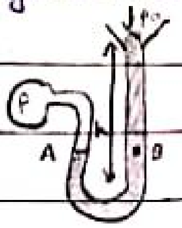
→ The space above mercury coloumn in the tube contain's only mercury vapour whose pressure p is so small that it can be neglected.

★ 8. What is guage pressure? how is a manometre used for measuring pressure differrence?

Sol: Gauge pressure: consider a point A at depth 'h' below the surface of the liquid open to the atmosphere. The total pressure at the point 'A' is greater than the atmospheric pressure by an amount ρgh where ρ is the density of the liquid.

Then total pressure $P = P_a + \rho gh$

$$P - P_a = \rho gh$$



The excess of pressure, $P - P_a$ at depth h is called a gauge pressure at that point.

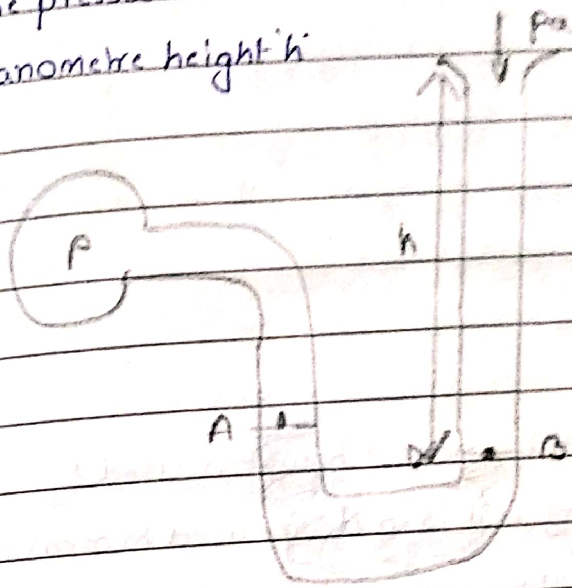
⇒ measurement of pressure differrence using manometre

→ An open-tube manometre is used for pressure differences

→ It consists of a U-tube containing a suitable liquid i.e, low density liquid such as oil for measuring small pressure differrence (or) a high density liquid for measuring larger pressure differrences

→ One end of the thermometre tube is open to the atmosphere and other is connected to the System whose pressure is to be is measured.

→ The pressure difference is directly proportional to manometre height 'h'



*9. What is hydrostatic paradox ?

Sol:- The absolute pressure of liquid at depth h depends upon the height of the liquid coloum but not the area of cross section (or) base (or) shape of vessel. The liquid pressure is same at all the points at same horizontal level. This is called hydrostatic paradox.

→ example:-

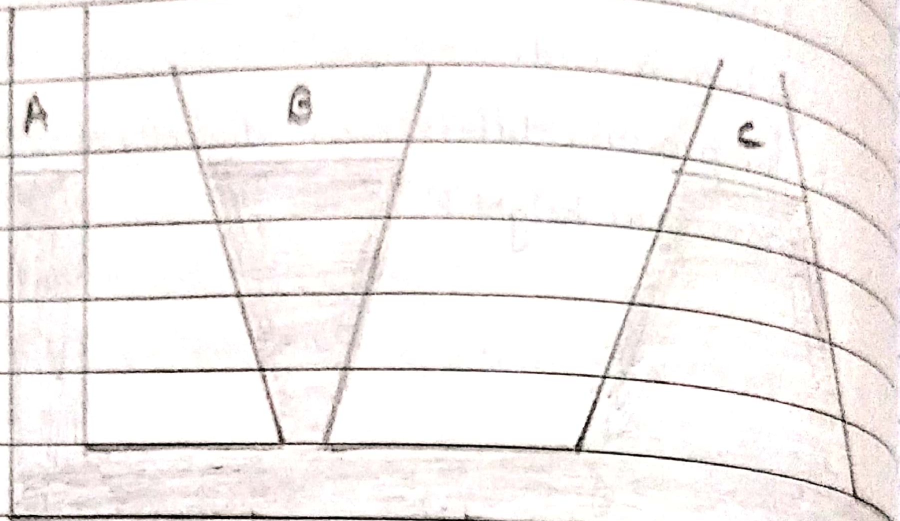
→ Consider a horizontal pipe is connected to different vessels B and 'c'

→ A is the horizontal pipe and B and c vessels are of different Area of cross section, shape and base

→ pour a liquid into the horizontal pump A then the liquid is spread into the vessels B and c equally.

→ The coloumn height of liquid in all vessels are equal.

→ This represent's a hydrostatic paradox.



★ 10. Explain venturi metre & how it is used?

Sol: venturimetre:- It is a device used to measure speed of the fluid flow

→ Consider a glass tube with different areas of cross section

→ A_1 is the crosssection at which the tube has wide diametre and A_2 is the Crosssection which as wide diametre

→ V_1 and v_2 are the velocities of fluid at A_1 and A_2

→ Attach a manometre to the glass tube Containing mercury and pressure P_m and having pressures P_1 and P_2 pressures

→ According to the bernoullis theorem

$P + \frac{1}{2}\rho v + \rho gh = \text{Constant}$

$\therefore P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho gh_1} = P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho gh_2}$

as $h_1 = h_2$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}(\rho_1 v_2^2 - \rho v_1^2)$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \quad \left\{ \text{as } P_1 - P_2 = \rho_m gh \right\}$$

$$\rho_m gh = \frac{1}{2}\rho[v_2^2 - v_1^2]$$

$$\frac{2\rho_m gh}{\rho} = v_2^2 - v_1^2$$

$\rightarrow$ as $A_1 v_1 = A_2 v_2$

$$A v_1 = a v_2$$

$$v_2 = \frac{A v_1}{a}$$

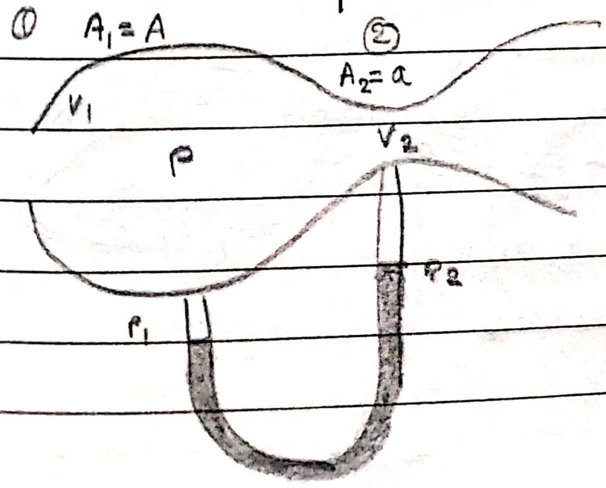
$$\therefore \frac{2\rho_m gh}{\rho} = \left[\frac{A v_1}{a}\right]^2 - v_1^2$$

$$\frac{2\rho_m gh}{\rho} = v_1^2\left[\left(\frac{A}{a}\right)^2 - 1\right]$$

$$\therefore v_1^2 = \frac{2\rho_m gh}{\rho\left[\left(\frac{A}{a}\right)^2 - 1\right]}$$

$$\therefore v_1 = \sqrt{\frac{2\rho_m gh}{\rho}} \cdot \frac{1}{\sqrt{\left(\frac{A}{a}\right)^2 - 1}}$$

$$v_1 = \sqrt{\frac{2\rho_m gh}{\rho}} \left[\left(\frac{A}{a}\right)^2 - 1\right]^{-1/2}$$



II LAQ's

★★ 1. State and proove bernoullis theorem?

Sol: Bernoulli's theorem :- Bernoulli's theorem states that when an incompressible, irrotational and non viscous fluid flow stedily then sum of pressure energy, kinetic energy and potential energy per unit volume remains constant at any point of path of the fluid flow.

⇒ proof:-

→ Consider a pipe with different areas of cross section

→ A_1 and A_2 are entering and ending of pipe

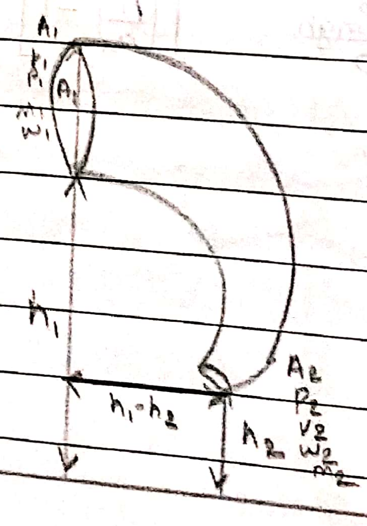
→ v_1 and v_2 are the velocities of fluid at entering and ending of pipe.

→ p_1 and p_2 are the pressures of fluid at entering and ending of pipe

→ m_1 and m_2 are the masses of fluids at entering and ending of pipe.

→ F_1, w_1 and F_2, w_2 are the force's and workdone at entering of pipe and ending of pipe

→ ①st point is at height h_1 and second point ②nd at height h_2



> As workdone = force x displacement

$$w = pAv\Delta t$$

$\therefore w_1 = P_1A_1v_1\Delta t$ and $w_2 = P_2A_2v_2\Delta t$

> work done due to pressure difference.

$$w_p = w_1 - w_2$$

$$\therefore w_p = P_1A_1v_1\Delta t - P_2A_2v_2\Delta t \quad -(1)$$

> work done due to gravitational force

$$w_g = mg[h_1 - h_2] \quad -(2)$$

> net work done :

$$w = w_p + w_g$$

$$\therefore w = P_1A_1v_1\Delta t - P_2A_2v_2\Delta t + mg[h_1 - h_2] \quad -(3)$$

> According to work energy theorem

w = change in kinetic energy.

$$w = \frac{1}{2}m[v_2^2 - v_1^2] \quad -(4)$$

{ According to law of continuity $v \propto 1/A$ as A decreases v increases }

$$\therefore v_2 >> v_1$$

> equating - (3) and (4)

$$P_1A_1v_1\Delta t - P_2A_2v_2\Delta t + mg[h_1 - h_2] = \frac{1}{2}m[v_2^2 - v_1^2]$$

→ here $m = Av\Delta t\rho$

$\therefore m_1 = A_1v_1\Delta t\rho_1$ and $m_2 = A_2v_2\Delta t\rho$

$\therefore$ but According to law of conservation of energy $m_1 = m_2$ and $\rho_1 = \rho_2$

$\frac{m_1}{\rho} = A_1v_1\Delta t$ and $\frac{m_2}{\rho} = A_2v_2\Delta t$

$$\Rightarrow A_1 v_1 \Delta t = \frac{m}{\rho}$$

$$A_2 v_2 \Delta t = \frac{m}{\rho}$$

$$\rightarrow P_1 \frac{m}{\rho} - P_2 \cdot \frac{m}{\rho} + mg[h_1 - h_2] = \frac{1}{2} m [v_2^2 - v_1^2]$$

$$\cancel{m}\left[\frac{P_1}{\rho} - \frac{P_2}{\rho} + g[h_1 - h_2]\right] = \frac{1}{2} \cancel{m} [v_2^2 - v_1^2]$$

$$\frac{P_1}{\rho} - \frac{P_2}{\rho} + gh_1 - gh_2 = \frac{1}{2} v_2^2 - \frac{1}{2} v_1^2$$

$$\frac{P_1}{\rho} + \frac{1}{2} v_1^2 + gh_1 = \frac{P_2}{\rho} + \frac{1}{2} v_2^2 + gh_2$$

$$\frac{P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1}{\cancel{\rho}} = \frac{P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2}{\cancel{\rho}}$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\therefore P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

hence, this is bernoulli's theorem.

12. Thermal properties of matter: Revision

I SAQ's :-

1. In what way analamous behaviour of water is advantageous to aquatic animals?

Sol: During winter in cold countries, water in ponds, lakes and river cools to 4°c due to decrease in atmospheric pressure and temperature. water has it's maximum density at 4°c. So the layer's of water remains at 4°c while the upper layer's further cool down to 0°c and freeze down into ice. Ice is a bad conductor of heat, so the lower layer of water protected against freezing. hence the aquatic animals can survive in lower layer's of water at 4°c.

2. pendulum clock's generally go fast in winter and slow in summer. why?

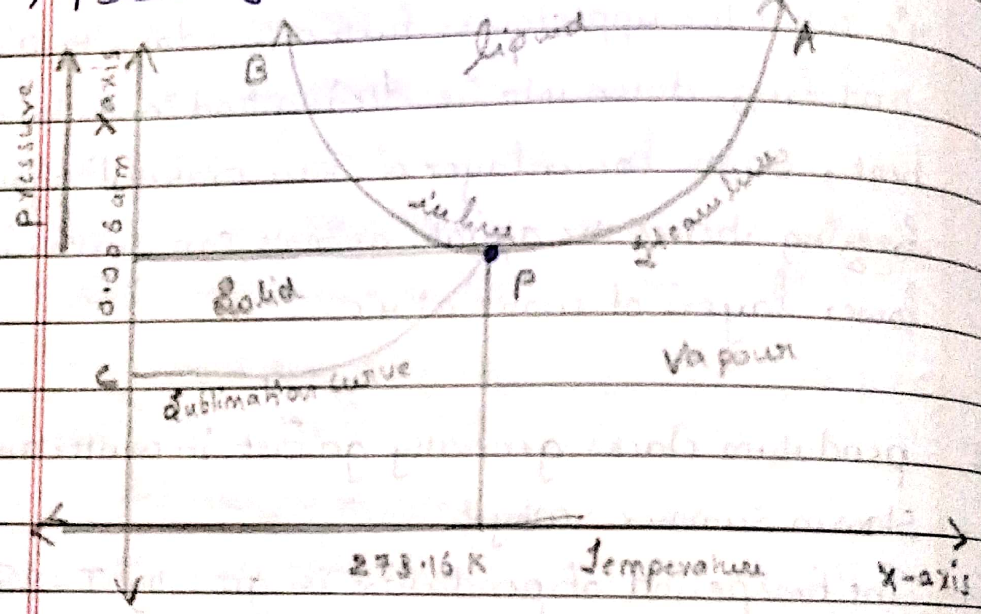
Sol: The time period of pendulum is given by $T=2\pi\sqrt{\frac{l}{g}}$ where l is the length of the pendulum. Thus T is directly proportional to $\sqrt{l}$. In summer T will increase as l increases. while in winter, T will decreases as l decreases. likewise, pendulum clock's go fast in winter and slow in summer.

3. write a short notes on triple point of water?

Sol: melting point:- The temperature at which the solid and liquid are at equilibrium is known as melting point boiling point :- The temperature at which liquid and gas are at equillibrium is known as boiling point.

Sublimation :- The temperature at which Solid's and gaseous states are at equillibrium position is called Sublimation

⇒ phase diagram :-



> The conditions in this diagram

pressure → 0.006 atm
temperature → 273.16 atm.

(i) PA is the steamline. It represents the variation of boiling point with pressure. As the pressure increases boiling point increases.

(ii) PB is the ice line. It represents the variation of melting point with pressure. As the pressure decreases the melting point decreases hence it indicates a negative slope.

(iii) pc is the submation line. It represents the variation of sublimation point with pressure. It is also known as hoarfrost line.

∴ The intersection of theese three lines is triple point.

⇒ triple point :- Triple point of water is a point in the phase diagram representing particular pressure and temperature at which solid, liquid and gaseous state exist in equilibrium state.

**4. what is celsius scale and farenheit scale and derive the relation between them?

Sol: Celsius scale :- A celsius scale has two fixed points
> upper fixed point
> lower fixed point.

→ At the lower fixed point the temperature is 0°c
→ At the higher fixed point the temperature is 100°c
→ Interaval between LFP and UFP is 100
→ They are divided into 100 equal parts

Farenheit scale :- A farenheit scale Also has two fixed points.

→ The temperature at lower fixed point is 32°F
→ The temperature at upper fixed point is 212°F
→ The UFP and LFP are divided into 180 equal parts

$$\therefore \frac{\text{Scale} - LFP}{UFP - LFP} = \text{constant}$$

⇒ Relation between celsius scale and farenheits scale.

→ celsius scale:

$$\frac{\text{Scale} - LFP}{UFP - LFP} = \text{constant}$$

Scale — C

LFP — 0°C

UFP — 100°C

$$\Rightarrow \frac{C-0}{100-0} \Rightarrow \frac{C}{100} = \text{constant} \quad — ①$$

→ farenheit scale:

Scale — F

LFP — 32°F

UFP — 212°F

$$\frac{F-32}{212-32} = \text{constant}$$

$$\frac{F-32}{180°} = \text{constant.} \quad — ②$$

> from the equation's ① and ②

$$\frac{C}{\cancel{100}_{5}} = \frac{F-32}{\cancel{180}_{9}}$$

$$\boxed{C = \frac{5}{9}[F-32]} \Rightarrow \boxed{F = \frac{9C}{5} + 32}$$

5. Explain conduction, convection and radiation?

Sol: Conduction :- The process of transfer of heat through a material without any actual movement of molecules (or) atom's but due to collission's between them is called Conduction.

ex:- In solid substances and in mercury heat flow's by the method of Conduction.

Convection :- The transmission of the energy from one part to another part in the body (or) system by the actual movement of the molecules (or) particle's is called Convection.

ex:- In fluid's (liquid's and gasses) heat flow's by the method of Convection

Radiation :- The transmission of heat from one place to another place without any material medium in the middle is called radiation

ex:- earth receives the heat from the Sun through the radiation.

II Laws :-

1. State newton's law of Cooling. State under which Condition's newton's law's are applicable?

Sol: Newton's law of cooling :- It States that the rate of loss of the heat of the hot body is directly proportional to the temperature difference between the hot body and Surroundings.

→ Consider a body with the high temperature.
→ There is a transfer of the heat energy from the hot body to the surrounding's in a time interval of dt

∴ amount of the heat energy transfered from the hot body to Surroundings is dq/dt

→ T is the temperature of the body.
→ T_s is the temperature of the Surroundings.
→ $T-T_s$ is the temperature difference between the hotbody and Surroundings
→ According to the newton's law of cooling

$$\frac{dq}{dt} \propto (T-T_s$$

$$\frac{dq}{dt} = -b\,(T-T_s) \quad \text{— (1)}$$

→ as Specific heat $s = \frac{dq}{mdT}$

$$dq = msdT$$

dividing with dT on both the sides

$$\frac{dq}{dt} = ms\,\frac{dT}{dt} \quad \text{— (2)}$$

→ from the equation's (1) and (2)

$$ms\,\frac{dT}{dt} = -b[T-T_s]$$

$$\frac{dT}{dt} = \frac{-b}{ms}[T-T_s]$$

as $\frac{-b}{ms} = k$

$$\boxed{\frac{dT}{dt} = k[T-T_s]}$$

where T is the rate of change of temperature.

⇒ condition's for which newton's law of cooling is applicable :-

(i) The temperature of the body should be uniformly distributed over it

(ii) The temperature difference is moderate.

(iii) Loss of heat is negligable by the conduction when it is only due to convection.

(iv) loss of the heat occurs in the streamlined flow of the air.

⇒ problem :- A body cools down from 60° → 50° in 5 min and to 40° in 8 min. find temperature of sorroundings.

Sol:- Given:-

Initially the body is at 60°c

cools to 50° in ⟶ 5 min

Cools to 40° in ⟶ 8 min

Step -(i) :- $\frac{dT}{dt} = K[T - T_s]$

$$\frac{60-50}{5} = K\left[\frac{60+50}{2} - T_s\right]$$

$$\frac{\cancel{10}\,2}{\cancel{5}} = K\left[\frac{110}{2} - T_s\right]$$

$$2 = K\left[55 - T_s\right]$$

$$\frac{2}{55 - T_s} = K$$

Step-(ii) $50°C \rightarrow 40°C$ in 8 min

$$\frac{dT}{dt} = K[T - T_s]$$

$$\frac{50-40}{8} = K\left[\frac{50+40}{2} - T_s\right]$$

$$\frac{\cancel{10}^{5}}{\cancel{8}_{4}} = \frac{2}{55-T_s}\left[45 - T_s\right]$$

$$\frac{5}{4} = \frac{2}{55-T_s}\left[45-T_s\right]$$

$$\frac{5}{8} = \frac{45-T_s}{55-T_s}$$

$$8[45-T_s] = 5[55-T_s]$$

$$360 - 8T_s = 275 - 5T_s$$

$$8T_s - 5T_s = 360 - 275$$

$$3T_s = 85$$

$$T_s = 85/3 \quad 28.33$$

$$\boxed{T_s = 28.33°C}$$

$\therefore$ The temperature of the Surroundings is $28.33°C$

★★ 2. Explain thermal Conductivity, and Coefficient of the thermal conductivity and problem of copper bar?

Sol:- Thermal Conductivity :- The process of transfer of the heat through a material without any actual movement of the molecules is called as Conduction. In this case collissions occur which is known as thermal Conductivity (or) Conduction

⇒ Coefficient of the thermal conductivity :-

→ Consider a rod with area of cross section A

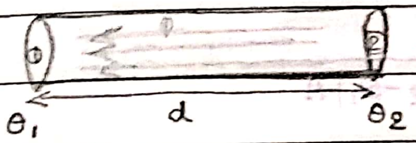
→ It is maintained with two different temperatures at the two ends.

→ Temperature of the first end is θ_1 and the temperature at second end is θ_2

→ If $\theta_2 > \theta_1$ then there will be a collission of the molecules from second end with the first end.

→ $\theta_2 - \theta_1$ is the temperature differrence in the rod.

→ d is the distance between the two ends of the rods.



→ here Q is the heat energy flowing. and Q/t is rate of flow of the heat energy.

→ $\theta_2 - \theta_1$ is the temperature differrence

therefore thermal conductivity can be defined as →

Coefficient of Thermal Conductivity :- The amount of the heat energy flowing per second through an area per unit temperature gradient.

Case (i) :- Rate of flow of heat energy is directly proportional to area of cross section.

$$Q/t \propto A \quad \text{---(1)}$$

Case (ii) :- Rate of flow of the heat energy is directly proportional to temperature difference

$Q/t \propto \theta_2 - \theta_1$ — ②

Case (iii) :- Rate of flow of the heat energy is inversly proportional to temperature difference

$Q/t \propto 1/d$ — ③

> from the equation's ①, ②, ③

$$\frac{Q}{t} \propto \frac{A(\theta_2 - \theta_1)}{d}$$

> proportionality is replaced with constant K = coefficient of thermal conductivity

$$K = \frac{Q}{t} \frac{d}{(\theta_2 - \theta_1) A}$$

where $\frac{\theta_2 - \theta_1}{d}$ is the temperature gradient

> units :- $\frac{W \cdot m}{K \cdot m^2} = W/Km$

dimensional formula :- $\frac{M^1 L^2 T^{-3}}{L \cdot K} = [M^1 L^1 T^{-3} K^{-1}]$

⇒ Problem :- A copper bar of the thermal conductivity 401 W/mK has 104°c in its one end and 24°c at other end and length of bar is 0.1 m and area of crossection is $1 \times 10^{-6} m^2$. Then rate of heat conduction alon- gtheir bar?

Sol:- Given :- $K = 401\ W/mK$

$\theta_2 = 104°c$

$\theta_1 = 24°c$

$d = 0.1m = 1\times10^{-1}m$

$A = 1\times10^{-6}m$

$\therefore Q/t \rightarrow ?$ {According to the Coefficient of thermal conductivity}

$$\therefore \frac{K \cdot A(\theta_2 - \theta_1)}{d} = Q/t$$

$$\frac{401 \cdot 1\times10^{-6}(80)}{1\times10^{-1}} = Q/t$$

$$401\times10^{-5}\times80 = Q/t$$

$$\therefore Q/t = 0.32080 \text{ Joules/Sec}$$

(or)

$$Q/t = 320.8\times10^{-3} \text{ Joules/Sec}$$

3. State boyles law, charles law, hence derive the ideal gas equation and which of the gas law is better for the thermometry and why?

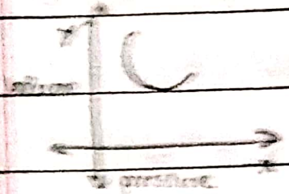
Sol:- → Boyle's law :- It states that at constant temperature the volume of a given mass of gas is inversly proportional to its pressure

$$p \propto \frac{1}{V}$$

$$pV = \text{Constant}$$

$$P_1V_1 = P_2V_2$$

> graphical representation of boyle's law :-



→ charles law :- It states that at constant temperature the volume of a given mass of gas is directly proportional to its absolute temperature.

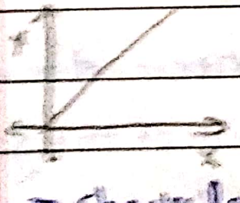
$V \propto T$

$\frac{V}{T}$ = constant

$\frac{V_1}{T_1}$ = constant and $\frac{V_2}{T_2}$ = constant

∴ $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

> graphical representation of charle's law :-



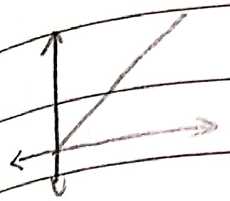
→ Charles law at constant volume :- It states that at the constant volume pressure of a given mass of gas is directly proportional to its absolute temperature

$P \propto T$

P/T = constant

∴ $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

> graphical representation of charles law at constant volume :-



Important note :- Ideal gas obey's boyle's law at low pressures and high temperatures.

⇒ Ideal gas :- The gas which obey's all gas laws at all the temperatures is known as the ideal gas.

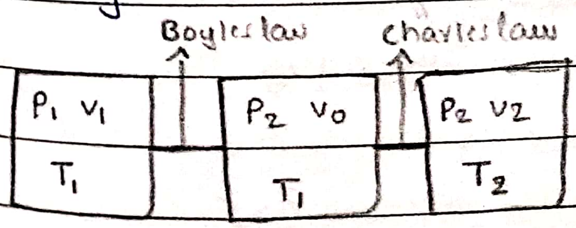
⇒ Ideal gas equation :-

> let us consider a container is filled with the one mole of ideal gas with initial pressure, volume and temperature P_1, T_1, V_1

> let $P_2 V_2, T_2$ are the final pressure, volume and temperature.

> After undergoing thermodynamical change in case of boyles law in Step - I

> And again undergoing thermodynamical change in case of charle's law in Step - II



→ Step - I :-

According to boyle's law at constant temperature

$$P_1V_1 = P_2V_0$$

$$V_0 = \frac{P_1V_1}{P_2} \quad \text{--- (1)}$$

→ Step - II :-

According to charles law at constant pressure

$$\frac{V_0}{T_1} = \frac{V_2}{T_2}$$

$$V_0 = \frac{V_2T_1}{T_2} \quad \text{--- (2)}$$

from the equations (1) and (2)

$$\frac{P_1V_1}{P_2} = \frac{V_2T_1}{T_2}$$

$$\frac{PV}{T} = \text{Constant}$$

$$\frac{PV}{T} = R$$

$\boxed{PV = RT}$ for one mole of gas

→ for the n moles of gas

$\boxed{PV = nRT}$

where R is the universal gravitational constant

R = 8.314J/mole.K

13. Thermodynamics revision

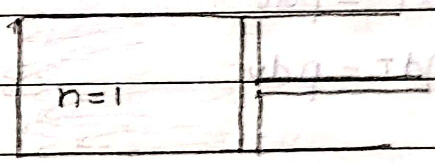
I SAQs:-

1. Derive the relation between two specific heat capacities of gas on the basis of first law of thermodynamics ? (or) $C_p - C_v = R$

Sol:- → let us consider a container with movable frictionless piston is filled with 1 mole of the ideal gas

→ let A be the area of the piston

→ P, V, T are pressure volume and temperature of gas



⇒ Step - (i) :- At constant volume :-

> dq is the amount of heat energy supplied to body

$\therefore (dq)_v = nC_v dT$

If $n = 1$

$(dq)_v = C_v dT$

> According to first law of thermodynamics

$dq = du + dw$

$(dq)_v = du + Pdv$ {as $pdv = 0$ at constant ~~volume~~}

$(dq)_v = du + 0$ { $du = C_v dT$ }

$\therefore \boxed{du = C_v dT}$ — ①

⇒ Step - (iii) :- At constant pressure :-

> $(dq)_p$ is the amount of heat energy supplied to body at constant pressure.

> $(dq)_p = nc_p dT$

If $n = 1$

$(dq)_p = C_p dT$

> According to first law of thermodynamics

$(dq) = du + dw$ {as $dw = pdv$}

$(dq)_p = du + pdv$

$C_p dT = C_v dT + pdv$ {from - ①}

$\therefore C_p dT - C_v dT = pdv$

$(C_p - C_v) dT = pdv$

> According to the ideal gas equation

$PV = nRT$

If $n=1$

$PV = RT$

$\therefore Pdv = RdT$ {as P and R are constant}

from equations ② and ③

$\cancel{dT}(C_p - C_v) = R\cancel{dT}$

$\boxed{C_p - C_v = R}$ //

hence proved.

★★★ 2. Obtain expression for the work done by the ideal gas during isothermal change?

Sol:- > By taking 1 mole of ideal gas in a container

> let $P_1 V_1$ are the initial volumes and pressures.

> as $dw = pdv$

Integration on both sides

$$\int dw = \int p dv$$

> If there is no function then the integration will be antiderivative.

$$w = \int_{v_1}^{v_2} p dv$$

> According to ideal gas equation :-

$$pv = nRT$$

for 1 mole of gas

$$pv = RT$$

$$p = RT/v$$

$$w = \int_{v_1}^{v_2} \frac{RT}{v} dv$$

$$w = \int_{v_1}^{v_2} \frac{dv}{v} RT$$

{as in isothermal ~~process R and T are~~ constant}

> from the formula $\int \frac{dx}{x} = \log_e x$

$$\therefore \quad w = RT \int_{v_1}^{v_2} \frac{dv}{v}$$

$$w = RT \log_e [v]_{v_1}^{v_2}$$

$$w = RT[\log_e v_2 - \log_e v_1]$$

$$\boxed{w = RT\left[\log_e\left[\frac{v_2}{v_1}\right]\right]}$$

> To convert logarithm to base e to 10 by multiplying constant 2.3026

$$\therefore \boxed{\omega = 2.3026\,(RT)\,\log_{10}\left(\frac{V_2}{V_1}\right)}$$

> As $P_1V_1 = P_2V_2$

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

$$\boxed{\omega = 2.3026\,(RT)\,\log_{10}\left[\frac{P_1}{P_2}\right]}$$

is work done in terms of pressure.

★★★ 3. Obtain an expression for the work done by ideal gas during adiabatic change?

Sol:- > The relation between pressure and volume is given by $pv^{\gamma} = K$ in adiabatic process

$$pv^{\gamma} = K$$

$$P_1 v_1^{\gamma} = k_1$$

$$P_2 v_2^{\gamma} = k_2$$

$$pv^{\gamma} = P_1 v_1^{\gamma} = P_2 v_2^{\gamma}$$

$$\therefore k = k_1 = k_2$$

> as work done

$$dw = pdv$$

Integration on both sides

$$\int dw = \int pdv$$

> If there is no function then the integration will be antiderivative

$$\omega = \int_{v_1}^{v_2} p\,dv$$

as $pv^\gamma = K$

$$P = K/v^\gamma$$

$$\therefore \omega = \int_{v_1}^{v_2} \frac{K}{v^\gamma}\,dv$$

$$\omega = K \cdot \int_{v_1}^{v_2} v^{-\gamma}\,dv$$

as $\boxed{\int x^n dx = \frac{x^{n+1}}{n+1}}$

$$\omega = K\left[\frac{v^{-\gamma+1}}{-\gamma+1}\right]_{v_1}^{v_2}$$

$$\omega = \frac{K}{1-\gamma}\left[v_2^{1-\gamma} - v_1^{1-\gamma}\right]$$

$$\omega = \frac{K}{1-\gamma}\left[v_2^{1-\gamma} - v_1^{1-\gamma}\right]$$

$$\omega = \frac{1}{1-\gamma}\left[Kv_2^{1-\gamma} - Kv_1^{1-\gamma}\right]$$

$$\omega = \frac{1}{1-\gamma}\left[K_2 v_2^{1-\gamma} - K_1 v_1^{1-\gamma}\right] \quad \left\{\text{as } K_1 = K_2 = K\right\}$$

$$\omega = \frac{1}{1-\gamma}\left[p_2 v_2^\gamma v_2^{1-\gamma} - p_1 v_1^\gamma v_1^{1-\gamma}\right]$$

$$\omega = \frac{1}{1-\gamma}\left[p_2 v_2^{\cancel{\gamma+1-\gamma}} - p_1 v_1^{\cancel{\gamma+1-\gamma}}\right]$$

$$\omega = \frac{1}{1-\gamma}\left[p_2 v_2 - p_1 v_1\right]$$

$$\omega = \frac{1}{\gamma-1}\left[p_1 v_1 - p_2 v_2\right]$$

> from the ideal gas equation

$$pv = nRT$$

→ for 1 mole

$$P_1V_1 = RT_1$$

$$P_2V_2 = RT_2$$

$$w = \frac{1}{\gamma-1}[RT_1 - RT_2]$$

$$w = \frac{R}{\gamma-1}[T_1 - T_2]$$

> for the n moles

$$\boxed{w = \frac{nR}{\gamma-1}[T_1 - T_2]}$$

//

*** 4. State and explain first law of thermodynamics?

Sol:- First law of thermodynamics :- First law of thermodynamics states that the amount of heat energy supplied to system is equal to the sum of change in internal energy and external work done.

$$dq = du + dw$$

where dq → amount of heat energy supplied to system

du → change in internal energy

dw → external work done

→ It is the special case of law of conservation of energy

⇒ Sign convention :-

> If the heat energy supplied to system dq is the positive and heat energy taken from the system is negative.

> If internal energy increases du is positive and if internal energy is decreased du is negative.

> work done by the system (dw) is positive and (du) is negative.

*5. Define two principle specific heat of a gas. which one is greater and why?

Sol:- ⇒ molar specific heat of a gas at constant pressure :-

The constant pressure the amount of heat energy required by one mole of gas to rise it's temperature by 1c

$$C_p = \frac{dq}{n(dT)}$$

$$\boxed{(dq)_p = n C_p dT}$$

Where $(dq)_p$ is amount of heat energy supplied to system at constant pressure.

⇒ Molar specific heat of a gas at constant volume:
At constant volume the amount of heat energy required by one mole of gas to rise its temperature by 1°C

$$C_v = \frac{dQ}{n(dT)}$$

$$\boxed{(dQ)_v = nC_v dT}$$

$(dQ)_v$ is the amount of heat energy supplied to the system at constant volume.

→ When heat is supplied to gas at constant volume, it is used only to increase. But if it heat is supplied to a gas at constant pressure in the two ways.
(i) To increase the temperature of gas {dU}
(ii) To do external work against the constant pressure hence more heat is suppleied to it.
→ Therefore for a given mass of gas for the same raise in temperature heat supplied to the gas at constant pressure is greater than heat supplied to constant volume. hence $C_p > C_v$

$$\therefore C_p - C_v = dw = pdv$$

Q6. Explain the qualitatively work done by the heat engine?

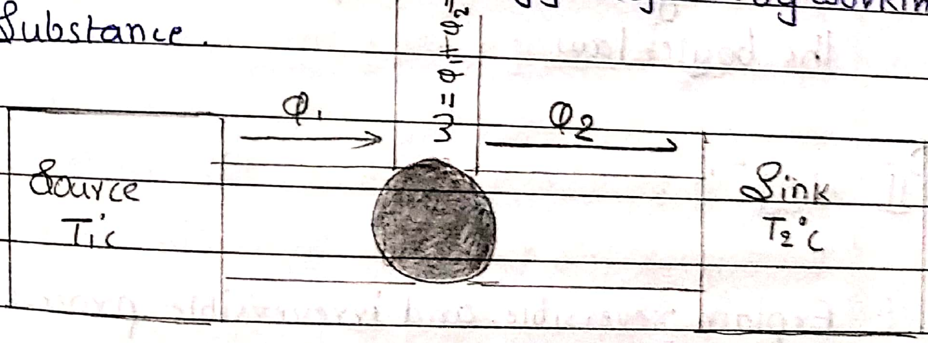
Sol: Heatengine:- heat engine is a device which converts the heat energy into mechanical energy.

=> essential parts of the heat engine:-
(i) Source
(ii) working Substance
(iii) Sink

-> Source:- It is maintained at high temperature $T_1°C$

-> working Substance:- In steam engines working substance is steam and in disel engine working substance is combination of fuel vapour and air.

-> Sink:- It is mainted at low temperature $T_2°C$ It absorb's the heat energy rejected by working substance.



> Consider a reservoir
> Q_1 is the heat rejected by the source which is absorbed by working substance
> Q_2 is the heat rejected by the working substance which is absorbed by sink
> T_1 and T_2 are temperature of source and sink.
> here work is done on the system.

**7. Compare isothermal and adiabatic process.

Sol: Isothermal process :- The process in which the pressure and volume changes at constant temperature is called isothermal process.

> Conditions to be fullfilled by isothermal process:
(i) The gas in this process must be a good conductor of heat
(ii) The gas used in the process obey's the boyle's law.

Adiabatic process :- The process in which pressure and volume changes in a thermally isolated system called as adiabatic process.

> Conditions to be fullfilled by adiabatic process
(i) The gas used in this process must be a bad conductor of heat.
(ii) The gas used in the process doesnot obey's the boyle's law.

II LAQ's :

*** 1. Explain reversible and irreversible process. Describe the working of carnot's engine. obtain an expression for efficiency.

Sol: Reversible process :- A process that can be retraced back in oppossite direction in such a way that the system passes through same state as in the direct process and return's to their original state known as reversible process.

→ Conditions for reversible process :-
(i) Changes that take place at infinitely slow rate
(ii) System must be equilibrium with the Surroundings
ex:- melting of ice

irreversible process :- A process that cannot be retraced back in the opposite direction is called irreversible process

ex:- (i) work done against friction
(ii) diffussion of gases.

⇒ Carnot's engine :- It is a heat engine and used to convert the heat energy into mechanical energy.

⇒ main parts of carnot's engine :-
(i) Source :- It is maintained at high temperature at T_1°c

(ii) working Substance :- The Steam engine working Substance is steam and in disel engine working Substance is the combination of full vapour and air

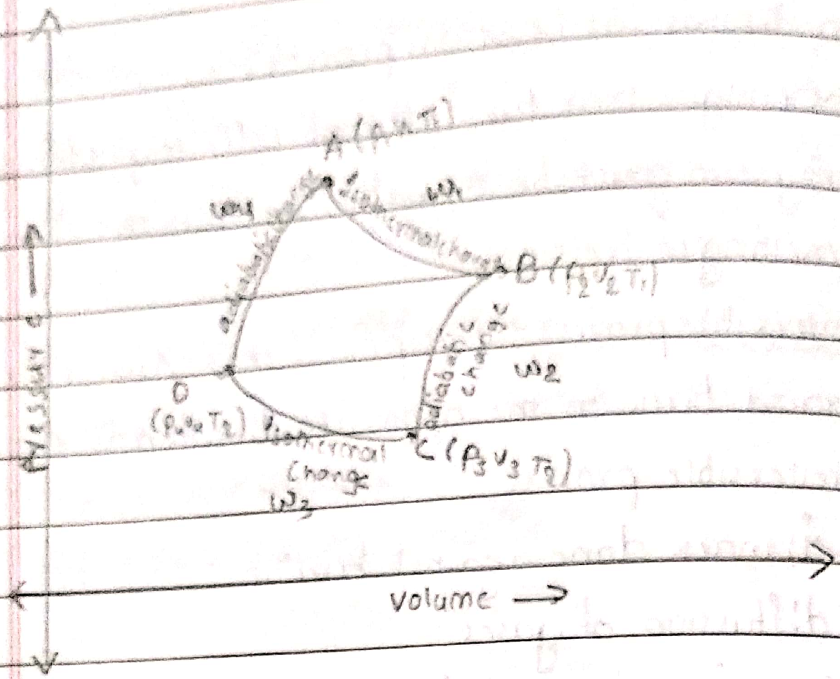
(iii) Sink :- It is maintained at low temperature T_2°c
It absorb's heat energy rejected by the working substance.

→ work done in Isothermal process :-

$$\omega = RT \log_e \left(\frac{V_2}{V_1}\right)$$

→ work done in adiabatic process :-

$$\omega = \frac{nR(T_2 - T_1)}{\gamma - 1}$$



⇒ Step :- (i) :-

> from AB it undergoes isothermal change.

$$Q_1 = W_1 = nRT_1 \log_e\left(\frac{V_2}{V_1}\right) \quad \text{— ①}$$

⇒ Step :- (ii) :-

> from BC, it undergoes adiabatic change

$$W_2 = \frac{nR}{\gamma - 1}\left[T_1 - T_2\right] \quad \text{— ②}$$

⇒ Step :- (iii) :-

> from CD, it undergoes isothermal change

$$Q_2 = W_3 = nRT_2 \log_e\left[\frac{V_3}{V_4}\right] \quad \text{— ③.}$$

⇒ Step :- (iv) :-

> from DA, it undergoes adiabatic change.

$$W_4 = \frac{nR}{\gamma - 1}\left[T_1 - T_2\right] \quad \text{— ④}$$

→ Then the network done is

$$w = w_1 + w_2 + w_3 + w_4$$

$$w = nRT_1 \log_e\left(\frac{V_2}{V_1}\right) + \cancel{\frac{nR}{\gamma-1}(T_1-T_2)} - nRT_2 \log_e\left(\frac{V_3}{V_4}\right) - \cancel{\frac{nR}{\gamma-1}(T_1-T_2)}$$

$$\therefore w = nRT_1 \log_e\left(\frac{V_2}{V_1}\right) - nRT_2 \log_e\left(\frac{V_3}{V_4}\right) \quad —④$$

→ efficiency of the Carnot's engine

$$\eta = \frac{w}{\phi_1} = 1 - \frac{\phi_2}{\phi_1}$$

$$\eta = 1 - \frac{\cancel{nR}T_2 \log_e\left(\frac{V_3}{V_4}\right)}{\cancel{nR}T_1 \log_e\left(\frac{V_2}{V_1}\right)}$$

$$\eta = 1 - \frac{T_2 \log_e\left(\frac{V_3}{V_4}\right)}{T_1 \log_e\left(\frac{V_2}{V_1}\right)} \quad —⑤$$

→ as $TV^{\gamma-1}$ = constant

→ from isothermal change :-

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\left(\frac{V_2}{V_3}\right)^{\gamma-1} = \frac{T_2}{T_1}$$

$$\left(\frac{T_2}{T_1}\right)^{1/\gamma-1} = \frac{V_2}{V_3}$$

> from the adiabatic change :-

$$T_2 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1}$$

$$\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = \frac{V_1}{V_4}$$

$$\therefore \eta = 1 - \frac{T_2 \cancel{\log_e (V_1/V_4)}}{T_1 \cancel{\log_e (V_1/V_4)}}$$

$$\boxed{\eta = 1 - \frac{T_2}{T_1}}$$

*** 2. State Second law of thermodynamics and how is the heat engine differrent from a refrigirator?

Sol: → planck's Statement:- no process is possible that it is to transfer the heat energy from low temperature body to high temperature without any unaid and it is impossible to convert the total heat into the work done.

→ Classius Statement :- It is highly impossible to transfer the heat energy from colder object to the hotter object without any external agency.

⇒ essential parts of heat engine :-

(i) Source

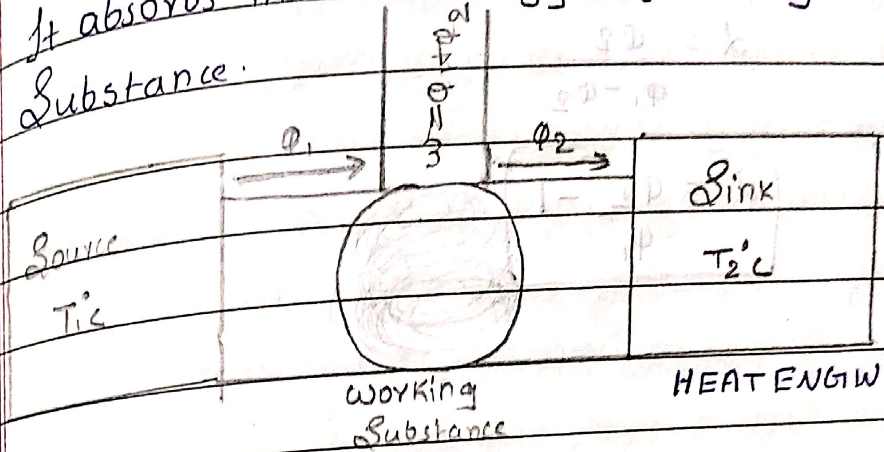
(ii) working Substance

(iii) Sink.

→ Source :- It is maintained at high temperature $T_1°C$

→ working Substance :- In Steam engines working Substance is Steam and in disel engine working engine is a combination of fuel vapour and air.

→ Sink :- It is maintained at low temperature. It absorbs the heat energy rejected by working Substance.



→ here work is the done on the System

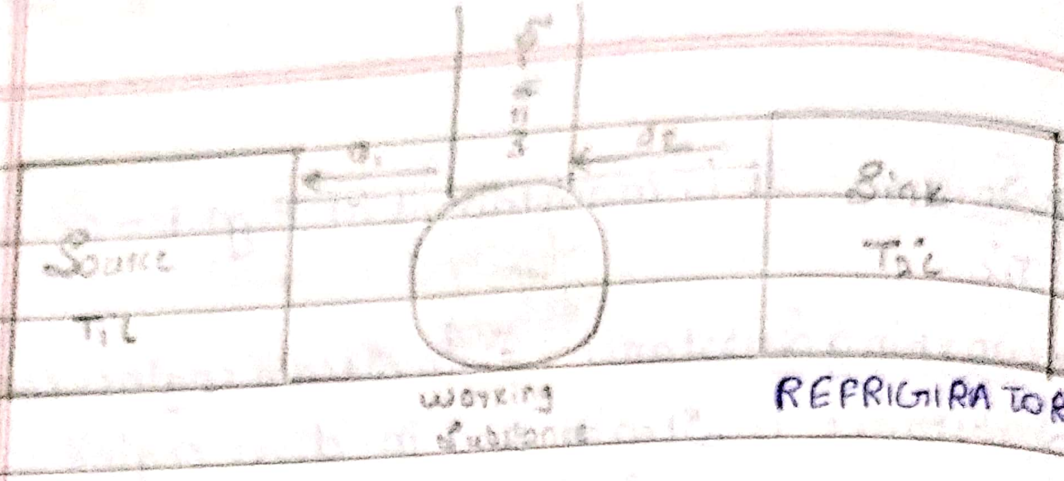
> consider a reservoir

> φ_1 is the heat rejected by working Substance which is absorbed by Sink

> T_1 and T_2 are temperature of Source and Sink

> This the heat engine

⇒ Refrigator :- In refrigator the working Substance extract the certain amount of heat energy from the Sink that is working of refrigirator is opposite to the working that of the heat engine.



REFRIGIRATOR

> (efficiency of performancy) (α) :-

$$\alpha = \frac{Q_2}{W}$$

$$\alpha = \frac{Q_2}{Q_1 - Q_2}$$

$$\boxed{\alpha = \frac{Q_2}{Q_1} - 1}$$

//

14. Kinetic theory of gases Revision

I SAQ's:-

Q1. How specific heat capacity of monoatomic, diatomic and polyatomic gasses can be explained on the basis of law of equipartition of energy?

Sol: ⇒ Law of equipartition of the energy:-
It States that each degree is associated with a freedom of $\left(\frac{1}{2}KT\right)$

> degrees of freedom :- The total number of the independent modes {ways} in such a way in the system posesses the energy is called as degrees of the freedom

they are represented

$$f = 3N - K$$

where

$N \rightarrow$ number of independent motions

$K \rightarrow$ number of independent restrictions.

(i) Monoatomic gas :- each atom of monoatomic gas has only 3 degrees of freedom. The total internal energy E (or) U of a gas is $U = \frac{3}{2}KT \times N_A$

$= \frac{3}{2}RT$ {as $KN_A = R$}

> The molar specific heat at constant volume is $C_V = \frac{du}{dT} = \frac{3}{2}R$ (1)

> for an ideal gas $C_P - C_V = R$,

$\Rightarrow C_P = C_V + R$

$\therefore c_p = \frac{5}{2}R$, $c_v = \frac{3}{2}R$ and $\gamma = \frac{c_p}{c_v} = \frac{5}{3}$

(ii) diatomic gas :- The diatomic gas molecule has 5 degrees of freedom and 3 - translational and 2 - rotational

> from the law of equipartitional energy

$$E \text{ (or) } U = \frac{5}{2} KT \times N_A$$

$$= \frac{5}{2} RT$$

$$C_v = \frac{du}{dT} = \frac{5}{2} R$$

> from $c_p - c_v = R \Rightarrow c_p = c_v + R \Rightarrow \frac{5}{2}R + R$

$$\therefore c_p = \frac{7}{2} R$$

$$\gamma = \frac{c_p}{c_v} = \frac{7}{5}$$

(iii) polyatomic gas :- The polyatomic gas molecule has 3 - translational, 3 - rotational and at least 1 (or) more vibrational modes according to law of equipartitional energy.

$$U = \left(\frac{3}{2}KT + \frac{3}{2}KT + fKT\right)N_A$$

$$\therefore c_v = (3+f)R, \quad c_p = (4+f)R$$

$$\therefore \gamma = \frac{c_p}{c_v} = \frac{4+f}{3+f}$$

Q. Four molecules of gas having speeds 1,2,3,4 Km/s find the rms speeds of the molecules.

Sol:- $V_{rms} = \sqrt{\frac{v_1^2+v_2^2+v_3^2+v_4^2}{4}}$

$V_{rms} = \sqrt{\frac{1+4+9+16}{4}}$

$= \sqrt{30/4}$ $\left(= \sqrt{\frac{15}{2}}\right)$

$= \sqrt{15/2}$

$= \sqrt{7.5}$

$= 2.74 \text{ Km/s}$

Q3. Explain the Kinetic interpretation of temperature?

Sol. > from the kinetic theory of gasses, average kinetic energy of gas molecule is given by

$K.E = \frac{1}{2}mv^2$

$\frac{1}{2}mv^2 = \frac{3}{2}KT$ {In terms of boltzmanns constant}

$\therefore \frac{1}{2}mv^2 \propto T$

∴ The average Kinetic energy of gas molecule is directly proportional to it's temperature

> If T=0 then

$\frac{1}{2}mv^2 = 0$

$\boxed{V=0}$ {body is at rest}

Therefore if the molecules of the body are at rest the temperature is known as absolute temperature

★★ 4. Show that average kinetic energy of gas molecule is directly proportional to the absolute temperature.

Sol:- > From the equation

$$p = \frac{1}{3}\left[\frac{Nm}{V}\right]v^2$$

$$pV = \frac{1}{3}\frac{Nm}{V}v^2$$

as $Nm = M$

$$pV = \frac{1}{3}mv^2$$

$$pV = \frac{2}{2} \times \frac{1}{3}mv^2$$

$$pV = \frac{2}{3} \cdot \frac{1}{2}mv^2$$

$$pV = \frac{2}{3}K.E$$

> as $pV = RT$

$$RT = \frac{2}{3}K.E$$

$$\therefore \boxed{K.E \propto T}$$ hence proved

★★ 5. find the ratio of rms speed of oxygen and hydrogen gas at same temperature?

Sol:

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{rms} \propto \frac{1}{\sqrt{m}}$$

$$\frac{V_{O_2}}{V_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}}$$

$$\frac{V_{O_2}}{V_{H_2}} = \sqrt{\frac{2}{32 \times 16}}$$

$$\frac{V_{O_2}}{V_{H_2}} = 1/4 \Rightarrow 1:4$$

Q6. If a gas have f degrees of freedom then find the ratio of c_p and c_v?

Sol: Given degree's of freedom $= f$

Total degrees of freedom $= Nf$

> T is the temperature of the system

$\therefore U = Nf \times \frac{1}{2} KT = \frac{f}{2} RT \quad \{\text{as } NK = RT\}$

> molar specific heat at constant volume

$$c_v = \frac{dU}{dT} = \frac{d}{dt}\left(\frac{f}{2} RT\right)$$

$$c_v = \frac{f}{2} R$$

> molar specific heat at constant pressure

$$c_p = c_v + R \Rightarrow \frac{f}{2} R + R$$

$$c_p = \left(1 + \frac{f}{2}\right) R$$

$$\gamma = \frac{c_p}{c_v} = \frac{\left(1+\frac{f}{2}\right)R}{\frac{f}{2}R} = 1 + 2/f$$

$$\therefore \boxed{\gamma = 1 + 2/f}$$